# Introduction to the distributed runoff model

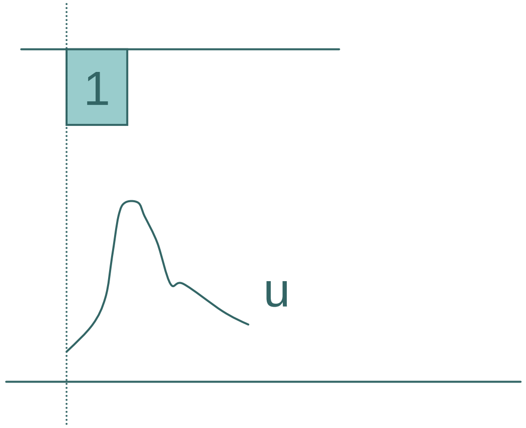
Toshiharu Kojiri
Kyoto University

# Runoff models

- Black box
  - Unit hydrograph
  - Time sequence
- Physical-based model
  - Lumped model
    - Storage function, Tank model
  - Distributed parameter model
    - Kinematic wave, etc.

# Unit hydrograph

$$Q(t) = \sum_{i=0}^{I} A \cdot r_e(t-i) \cdot u(i)$$




*Q(t):* runoff discharge
*A*: area
*$r_e$(t):* allowable rainfall intensity
*u*(*t*): unit hydrograph (runoff discharge)

Linear characteristics are strongly kept

-Independence (Same runoff in the same basin)
-Proportion (Rainfall intensity generates the proportional discharge)
-Linearity (Long-term rainfall is divided into units)

# Time sequence

- Linear regression

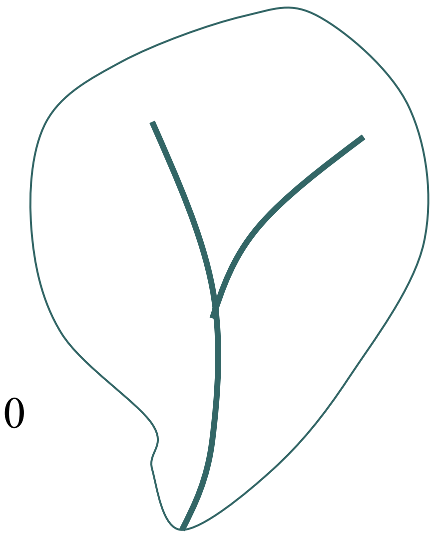
$$Q_1(t+3) = a_{10}(t)Q_1(t) + a_{11}(t)Q_1(t-1) + \Lambda \; + a_{1T}(t)Q_1(t-T)$$
$$+ a_{20}(t)Q_2(t) + \Lambda \; + a_{2T}(t)Q(t-T) + \Lambda \; + b_{10}(t)r_1(t)$$
$$+ b_{11}(t)r_1(t-1) + \Lambda$$

- ARMA($p$,$q$) ,

$$\mathbf{Y}_t - \Phi_1 \mathbf{Y}_{t-1} - \mathrm{L} \; - \Phi_p \mathbf{Y}_{t-p} = \mathbf{Z}_t + \Theta_1 \mathbf{Z}_{t-1} + \mathrm{L} \; + \Theta_q \mathbf{Z}_{t-q}$$

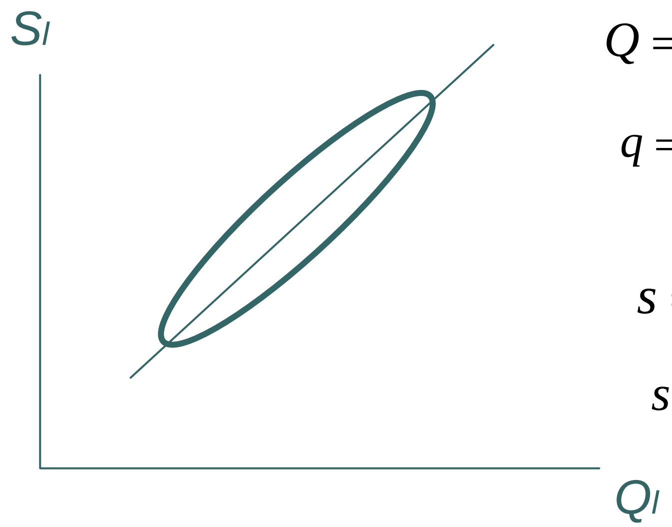
- ARIMA($p$,$r$,$q$)

# Storage function



$$\frac{dS_l}{dt} = I(t) - O(t) = f \cdot r_e(t) - q_l(t) \qquad f\int_{t1}^{t2} r_e(t)dt - \int_{t1}^{t2} q_l(t)dt = 0$$

$$S_l(t) = KQ_l^p(t)$$




$$Q = Av = \frac{A}{n}R^{2/3}I^{1/2} \qquad R \approx h$$

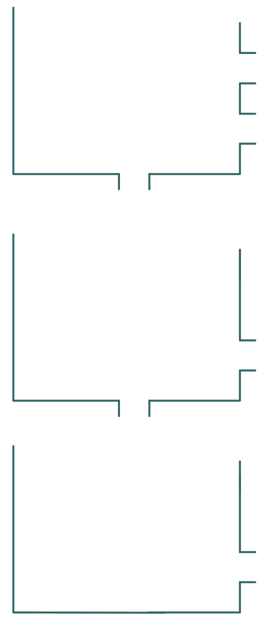
$$q = \frac{Av}{B} = \frac{1}{n}h^{5/3}I^{1/2}$$

$$s = S / A = h$$

$$s = h = k \cdot q_l^p = k\left(\frac{1}{n}h^{5/3}I^{1/2}\right)^p$$

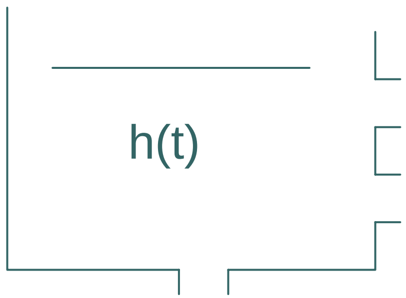
$$= k\left(\frac{I^{1/2}}{n}\right)^p \left(h^{5/3}\right)^p \qquad \frac{5}{3}p = 1$$

# Tank model



$$h(t) \geq H1$$
$$Q1(t) = \lambda 1(h(t) - H1)$$
$$Q2(t) = \lambda 2(h(t) - H2)$$
$$Q3(t) = \lambda 3h(t)$$

$$H1 \geq h(t) \geq H2$$
$$Q1(t) = 0$$
$$Q2(t) = \lambda 2(h(t) - H2)$$
$$Q3(t) = \lambda 3h(t)$$




$$H2 \geq h(t)$$
$$Q1(t) = 0$$
$$Q2(t) = 0$$
$$Q3(t) = \lambda 3h(t)$$

$$A \cdot r_e(t) - (Q1(t) + Q2(t) + Q3(t)) = \frac{dh(t)}{dt}$$

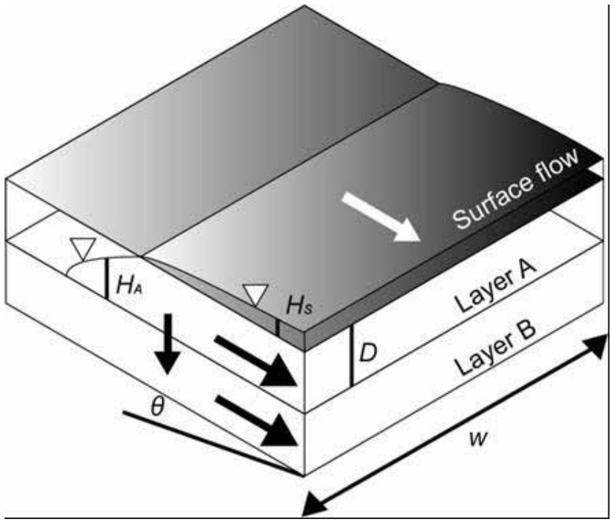
# Flood runoff

Energy equation:

$$\frac{1}{g}\frac{\partial v}{\partial t} + \frac{v}{g}\frac{\partial v}{\partial x} + \frac{\partial h}{\partial x} = S_e - S_f$$

Continuous equation

$$\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} = r(t, x)$$




Steady $$\frac{\partial v}{\partial t} = \frac{\partial h}{\partial t} = 0$$

Uniform $$\frac{\partial v}{\partial x} = \frac{\partial h}{\partial x} = 0$$

*A* is cross-sectional flow area [m2],
*q* is discharge [m3/s], *t* is time [s],
*x* is longitudinal distance along a channel or surface [m], and
*r* is lateral inflow per unit length of flow [m3/m.s]

# Flood runoff II

- Dynamic wave

$$\frac{1}{g}\frac{\partial v}{\partial t}+\frac{v}{g}\frac{\partial v}{\partial x}+\frac{\partial h}{\partial x}=S_e-S_f$$

- Diffusion wave

$$\frac{\partial h}{\partial x}=S_e-S_f$$

- Kinematic wave

$$0=S_e-S_f$$

Se: channel gradient

Sf: friction loss

# Kinematic wave I

Continuous equation

$$\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} = r(t, x)$$

Momentum equation

$$q = f(x, h)$$

*A* is cross-sectional flow area [m2],
*q* is discharge [m3/s], *t* is time [s],
*x* is longitudinal distance along a channel or surface [m], and
*r* is lateral inflow per unit length of flow [m3/m.s]

# Kinematic wave II

## -Characteristics function I-

$$q = f(x,h) \qquad (1)$$

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r(t,x) \qquad (2)$$

Differential relationship

$$\frac{dh}{dt} = \frac{\partial h}{\partial t} + \frac{\partial h}{\partial x}\frac{dx}{dt} \qquad (3)$$

From eq. (1),

$$\frac{\partial q}{\partial x} = \frac{\partial f}{\partial h}\frac{\partial h}{\partial x} + \frac{\partial f}{\partial x}$$

And then replacing in eq. (2)

$$\frac{\partial h}{\partial t} + \frac{\partial f}{\partial h}\frac{\partial h}{\partial x} + \frac{\partial f}{\partial x} = r(t,x) \quad \Rightarrow \quad r(t,x) - \frac{\partial f}{\partial x} = \frac{\partial h}{\partial t} + \frac{\partial f}{\partial h}\frac{\partial h}{\partial x} \qquad (4)$$

Comparing eq. (3) and eq. (4),
calculation procedures on characteristic function method are obtained.

# Kinematic wave III

## -Characteristics function II-

Water depth: $$r(t,x) - \frac{\partial f}{\partial x} = \frac{dh}{dt}$$

$$\frac{\partial f}{\partial x} \cong 0$$ For no infiltration zone

r: effective rainfall

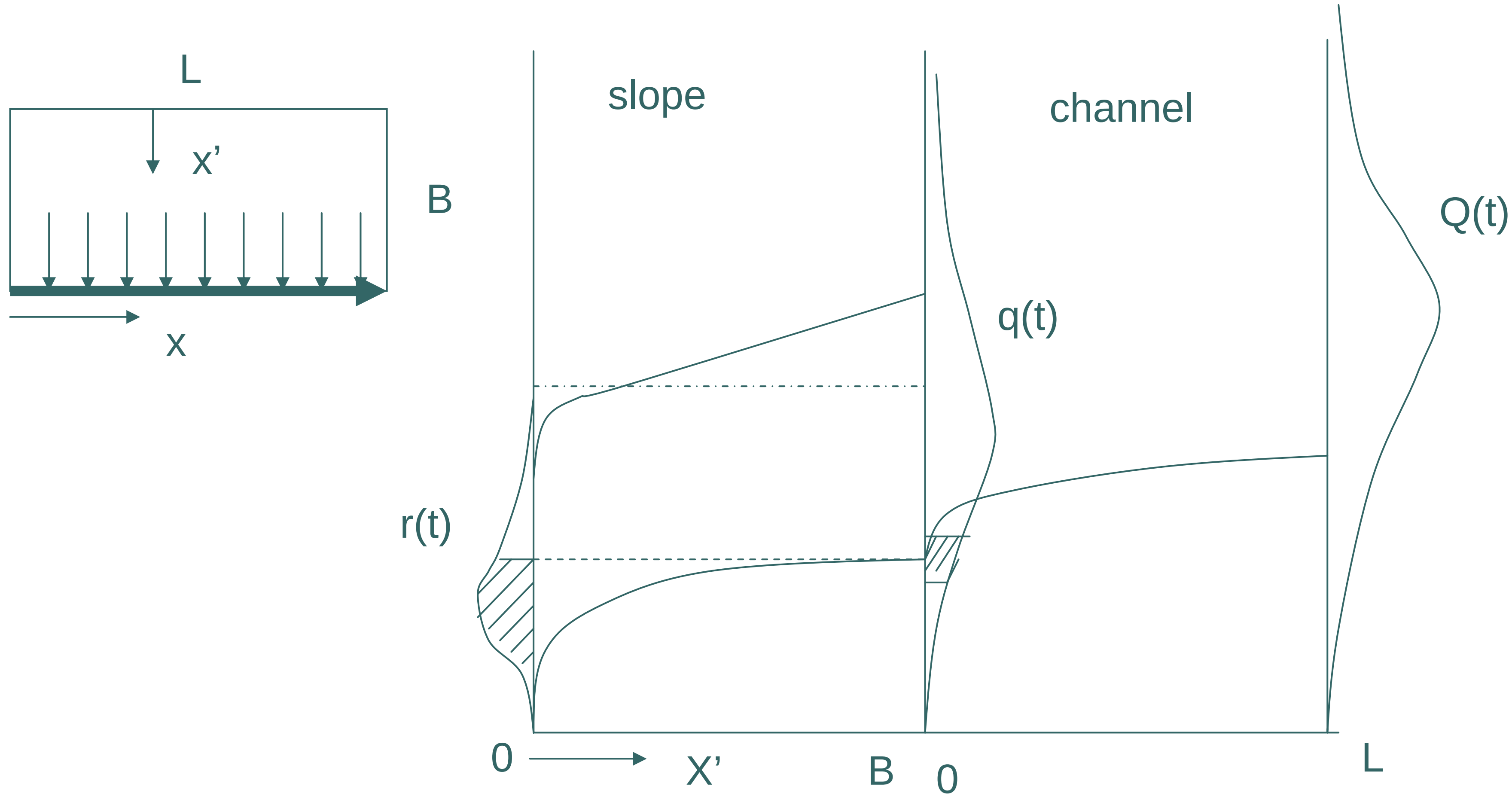
Velocity: $$\frac{\partial f}{\partial h} = \frac{dx}{dt}$$

In the case of Manning velocity $$q = vh = \frac{1}{n} R^{2/3} I^{1/2} h = \frac{1}{n} h^{5/3} (\sin\theta)^{1/2}$$

$$\frac{dx}{dt} = \frac{\partial q}{\partial t} = \left(\frac{1}{n}(\sin\theta)^{1/2}\right) \frac{\partial\left(\frac{1}{n}(\sin\theta)^{1/2}\right) h^{5/3}}{\partial t} = \frac{\partial \alpha h^m}{\partial t}$$

$$= \alpha m h^{m-1}$$

# Schematic diagram showing propagation states



# Kinematic wave IV

## -Characteristics function III-

In the case of r(t)>0

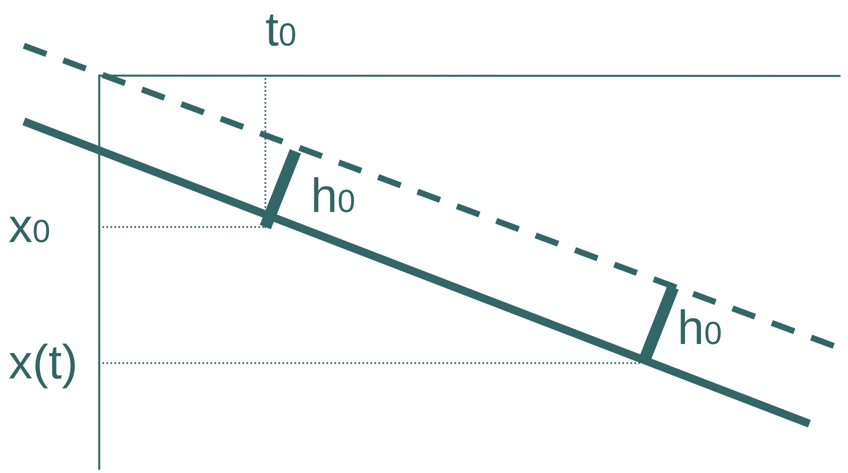
$$h(t) = r(t - t_0) + h_0$$

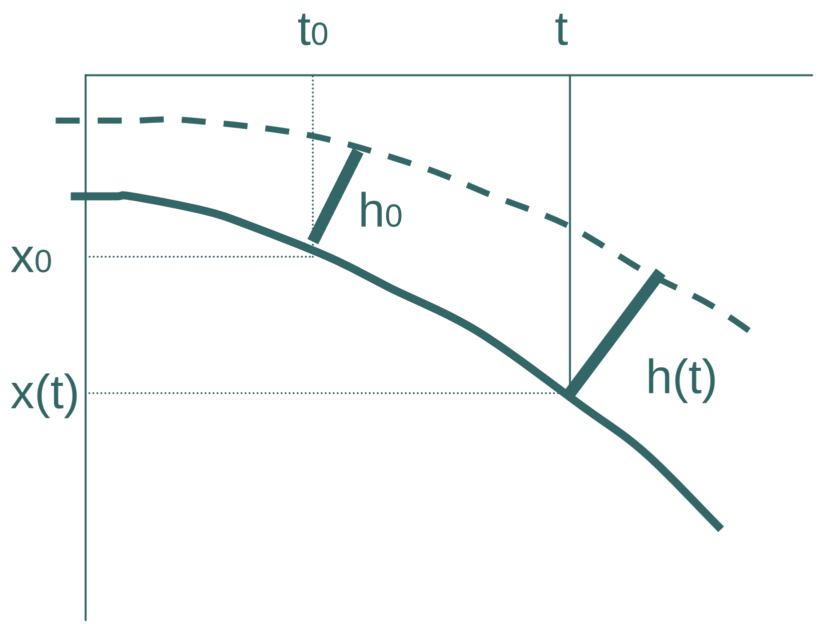
$$\frac{dx}{dt} = \alpha m \{ r(t - t_0) + h_0 \}^{m-1} \quad x(t) = \alpha \{ h(t)^m - h_0^m \} / r(t_0) + x_0$$

In the case of r(t)=0

$$\frac{dh}{dt} = 0 \quad h(t) = h_0$$

$$\frac{dx}{dt} = \alpha m h_0^{m-1} \quad x(t) = \alpha m h_0^{m-1} (t - t_0) + x_0$$

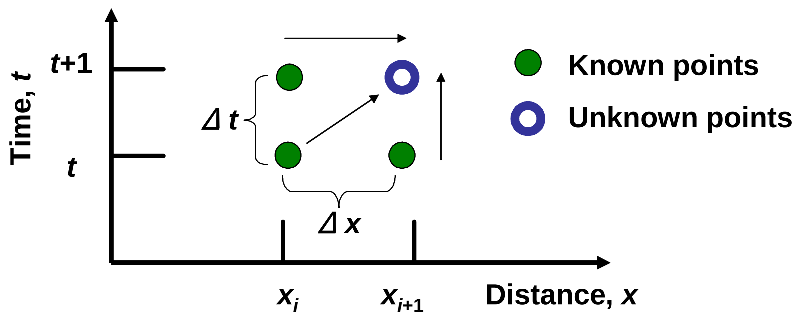


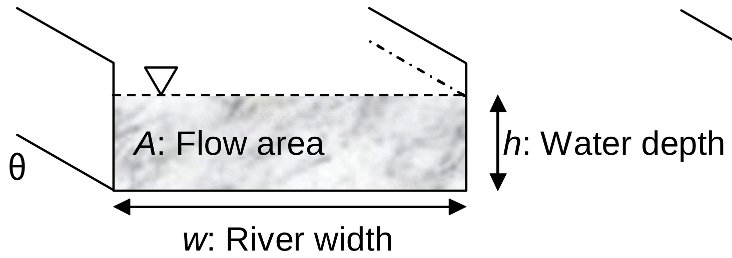
# Kinematic wave V

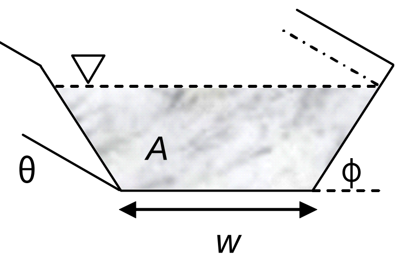
## -Characteristics function IV-

Differential analysis:




Calculation development:




$$q = \frac{\sqrt{\sin(\theta)}}{n} A^{5/3} (w + 2h\sqrt{1+z^2})^{-2/3}$$

$$q = \frac{\sqrt{\sin(\theta)}}{n} A^{5/3} (w + 2h)^{-2/3}$$

# Distributed runoff model

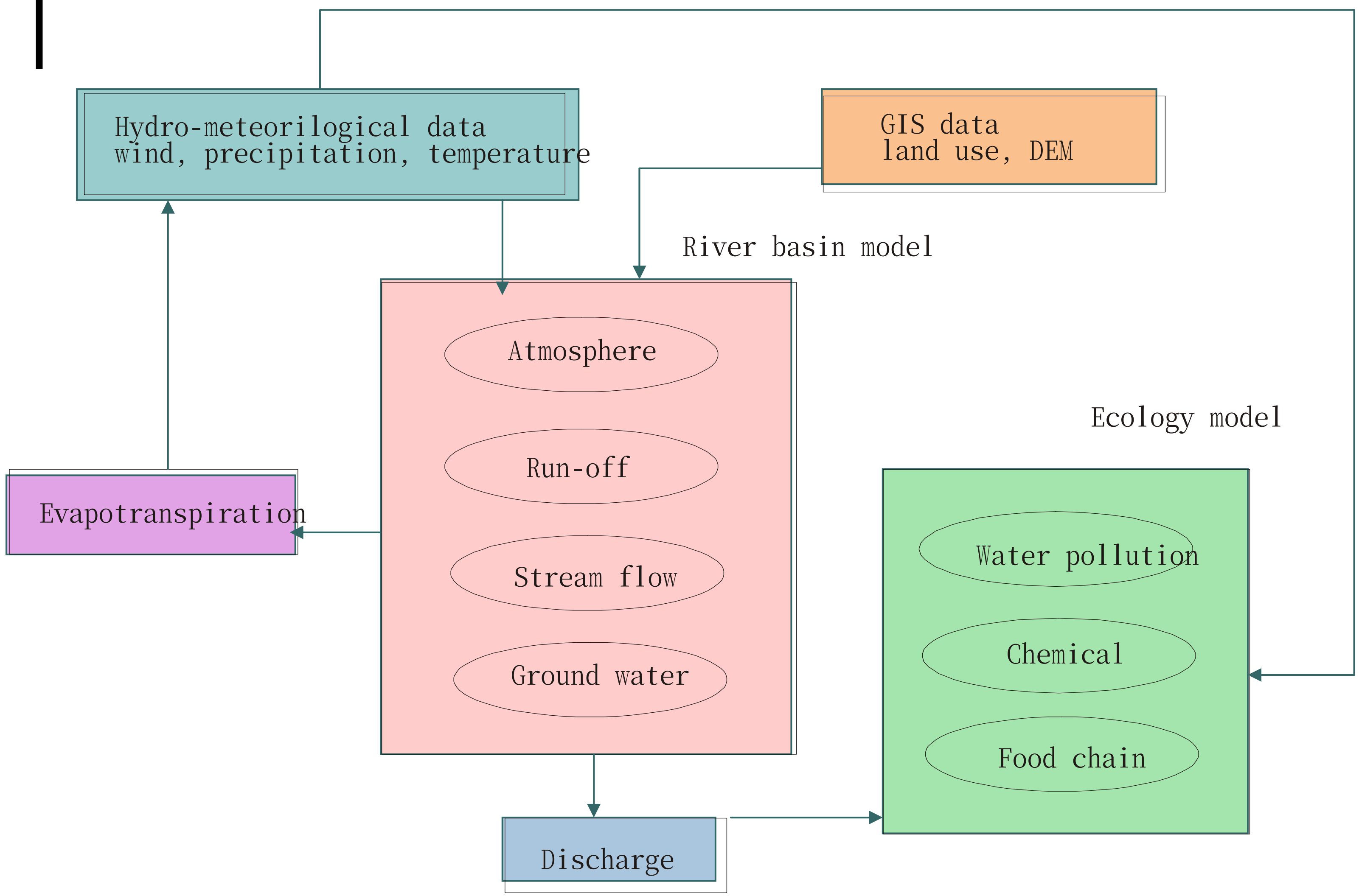
River basin

- The basin boundary is determined with DEM.
- The basin is divided to uniform square grids (1 km x 1 km).
- The subsurface has four uniform layers, (A, B, C, D).
- Land use data is assumed to be fixed except for the paddy field type.

Hydrological processes

- The heat balance method is used to calculate the daily potential evaporation.
- Ground heat flux is approximated by using an empirical equation that based on the cyclic fluctuation of the ground temperature
- Parameterization of the four layers is based on the land use type.
- The calculated evaporation rate is used in the mass balance equation of the surface and subsurface layers.
- The kinematic wave method is used for routing the runoff and the river flow rate.

# Water circulation



# Evapotranspiration

Heat balance $IR = \sigma T^4 + HS + lE$

- Input radiation； $IR = (1 - ref)SR + LR$
- Bulk formula ; $HS = C_p \rho C_H U(T_s - T)$
- Laterant heat ; $lE = l\rho C_E U(q_s - q_h)$

Thornthwaite

$$Ep = 0.553 D_0 (10 T_i / H)^a$$

$$H = \sum_{i=1}^{12} (T_i / 5)^{1.514}$$

$$a = (0.675 H^3 - 77.2 H^2 + 17920 H + 492390) \times 10^{-6}$$

# Interception

interception caused by residential buildings

$$I = D + VEt$$

*I* : the interception loss,
*D* : the interception storage depth,
*V* : the ratio of building's surface area to its projected area on the ground,
*E* is the evaporation rate.

# Snowfall-snowmelt

Judgment for snowfall $$T_c = (11.01 - 1.5e_a) + 273.15$$

Equivalent water content $$w_{eqn} = r_s - w_{cn}$$

$$w_{cn} = 0.05 r_s \qquad T_{sn} = 273.15$$

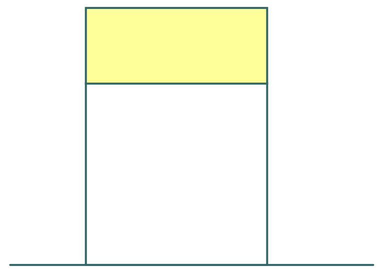
$$w_{cn} = 0 \qquad T_{sn} = T_a$$

$$for\ Ta = 273.15$$

Compaction process $$\tilde{\rho}(t) = \rho(t) + \frac{w_{eq}(t)}{\eta}\rho(t)\Delta t$$

$$\eta = 3.44 \times 10^6 A_T \exp\left(0.0253\rho(t) - 0.0958 T_s(t)\right) \quad Ta \le Tc$$

$$A_T = 10^{-(0.179(T_a(t) - 273.15) + 2.15)} + 0.0015 \qquad A_T = 1.0$$



Snow depth $$H_{ss}(t) = \frac{w_{eq}(t)}{\rho_n(t)}$$

Calorie for snow melt $$Q_m = 86400\left(R\downarrow + Q_a - \sigma T_s^{\ 4} - H - lE - Q_s\right) + Q_r$$

$$Q_r = c T_a r_s \qquad Q_s = 1.16 \times 10^{-5} l_F$$

# Ground water

## Subsurface unsaturated flow

$$C\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial z}\left[K(\Psi)\left(\frac{\partial \Psi}{\partial z}+1\right)\right]$$

$$S_t = \frac{\theta-\theta_r}{\theta_s-\theta_r} = \left[\frac{1}{1+\left(\Psi/\Psi_d\right)^{\alpha}}\right]^{\lambda/\alpha}$$

$$K = K_s S_t^{\eta}$$

## Saturated groundwater flow

$$\frac{1}{2}\left(K_x\frac{\partial^2 h^2}{\partial x^2}+K_y\frac{\partial^2 h^2}{\partial y^2}\right) = S\frac{\partial h}{\partial t} - R(t)$$

*C* : the specific water capacity,
***ψ*** (m) : the water potential head,
*K* : the hydraulic conductivity (m/sec),
*z* (m) : the vertical coordinate,

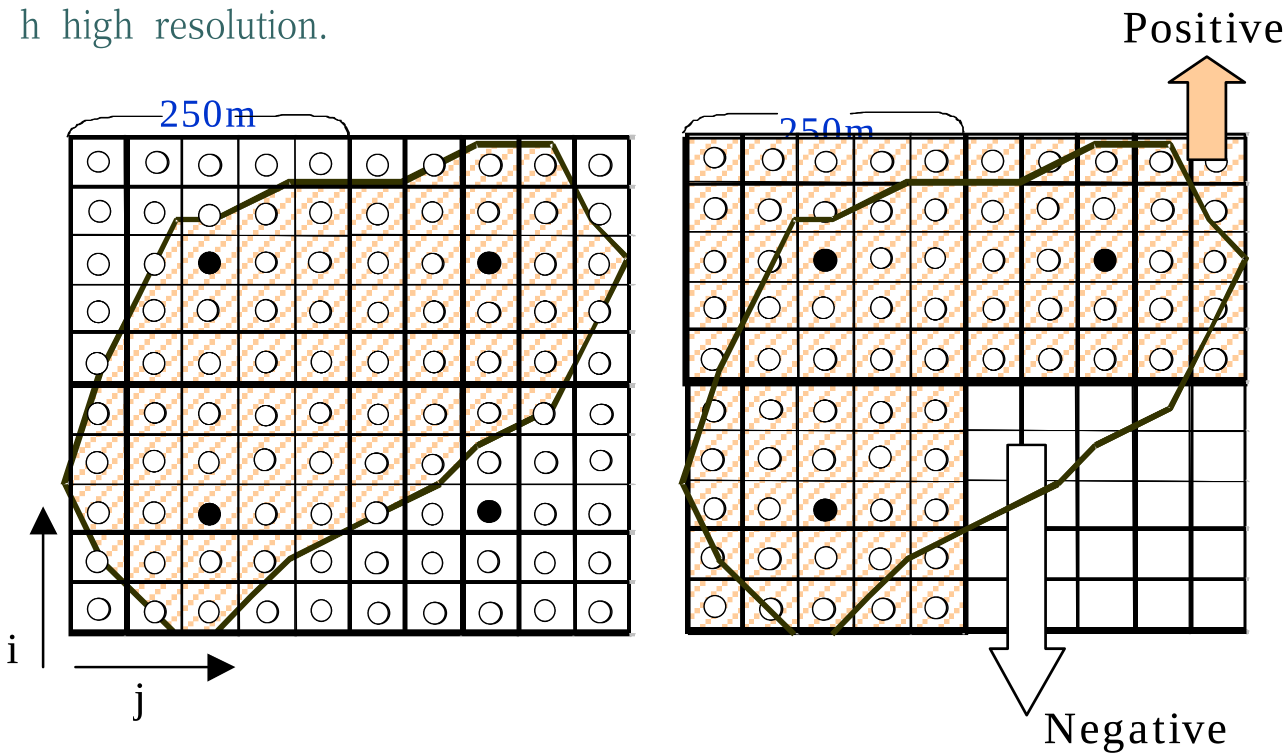
*St* : the effective saturation,
***Θ****r* : the residual water content,
***θ****s :* the saturated water content,
***α*** and ***λ*** *:* the van-Genuchten parameters,
***Ψ****d* : the displacement pressure head,
*Ks* : the saturated hydraulic conductivity,
***η*** : the exponential soil parameter,
*h :* the groundwater level,
*S* : the specific yield,
*R :* the volume of recharge per unit time per unit aquifer area,
*x* and *y* : the displacement coordinates

## Linear Storage Method

$$\frac{dS}{dt} = I - O \qquad O = kS$$

# Optimized mesh structure

Mesh structure must be optimized because DEM provides smaller grid data wit h high resolution.




$$\sum_{k=1}^{n} [NEGATIVE(A_k)^2 + POSITIVE(A_k)^2] \rightarrow Min$$

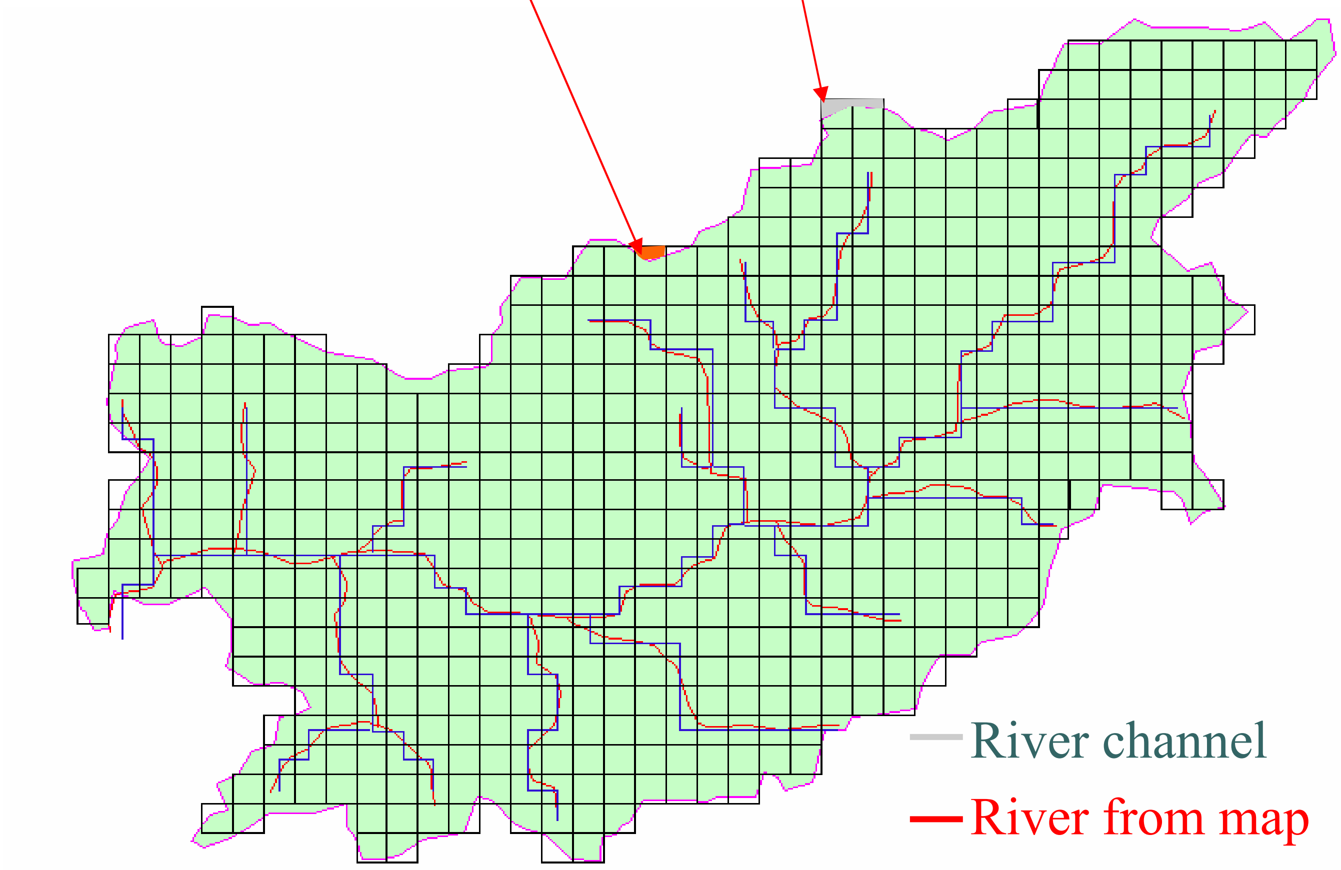
$NEGATIVE(A_k)^2$: non-countable area of river basin in mesh

$POSITIVE(A_k)^2$: countable are out of river basin in mesh

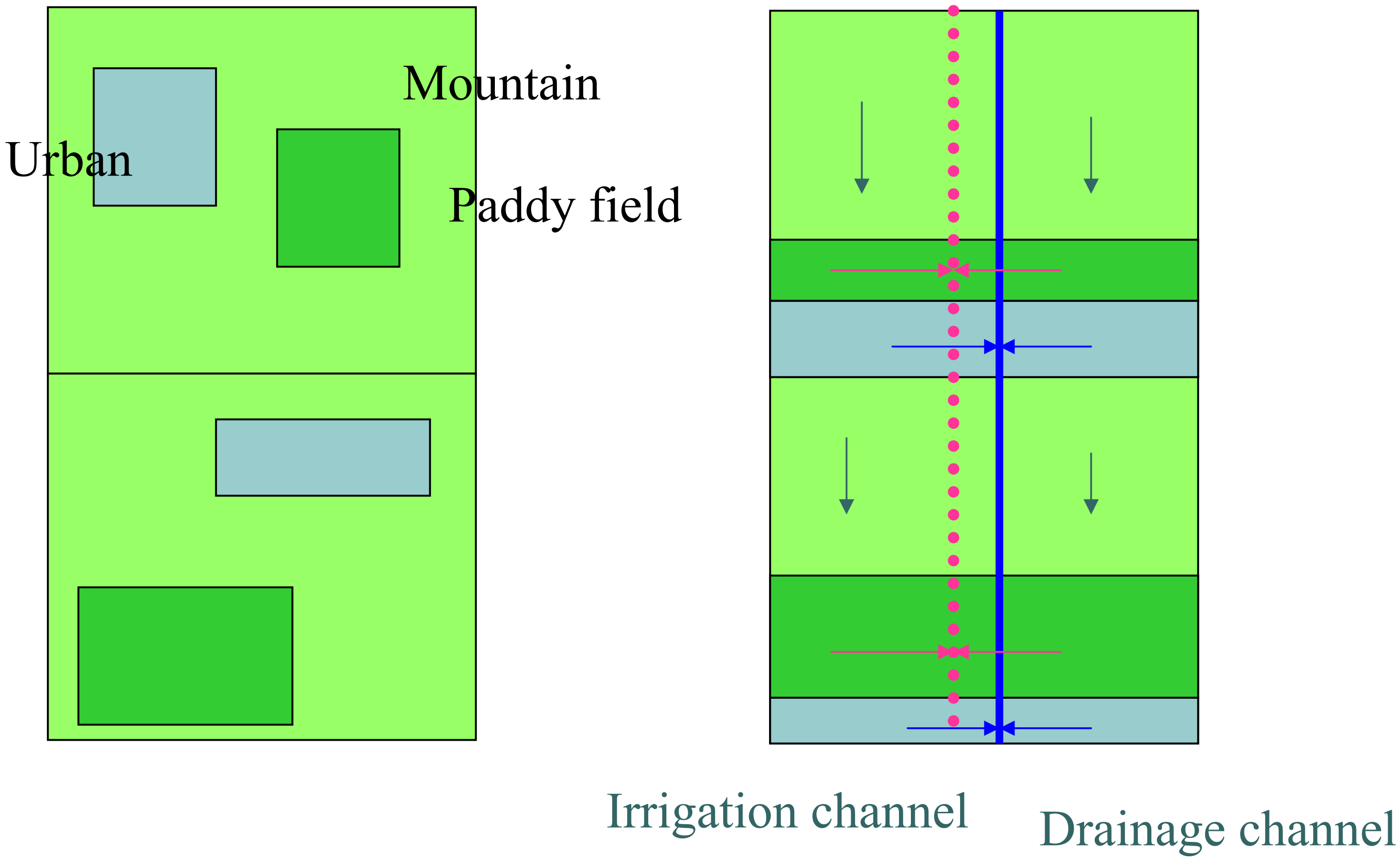
# Optimized result for mesh structure

$\Sigma\ (\mathrm{NEGATIVE}(A_k)^2 + \mathrm{POSITIVE}\ (A_k)^2) \rightarrow \mathrm{Min}$

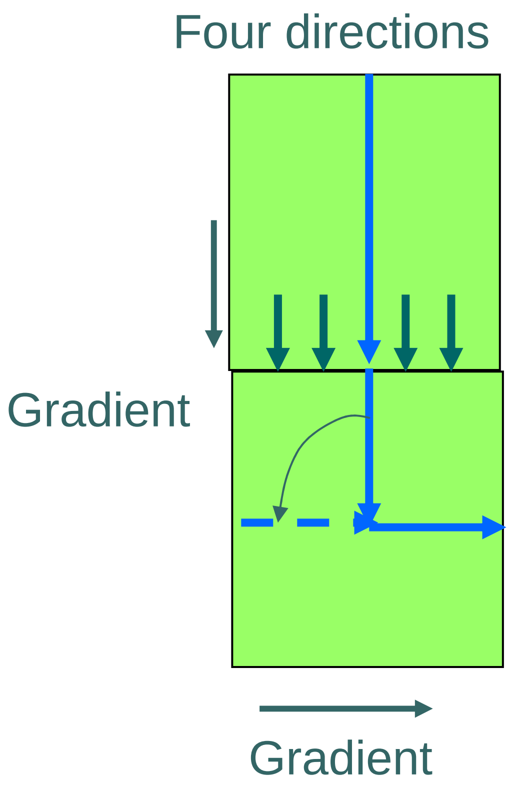
**Real Area** **Mesh Number**

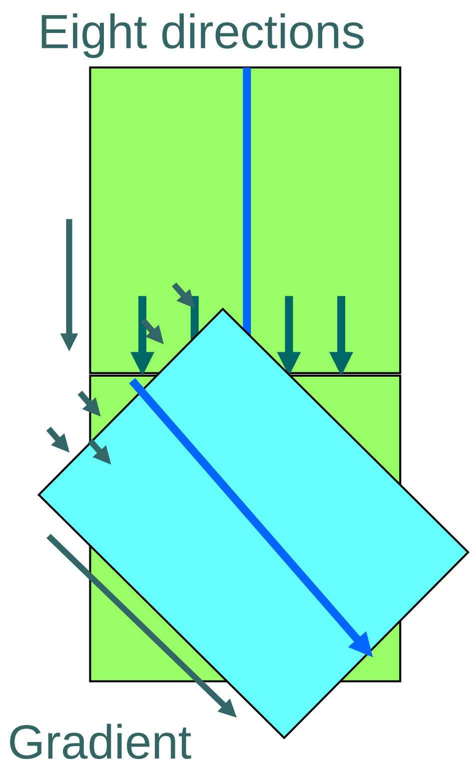


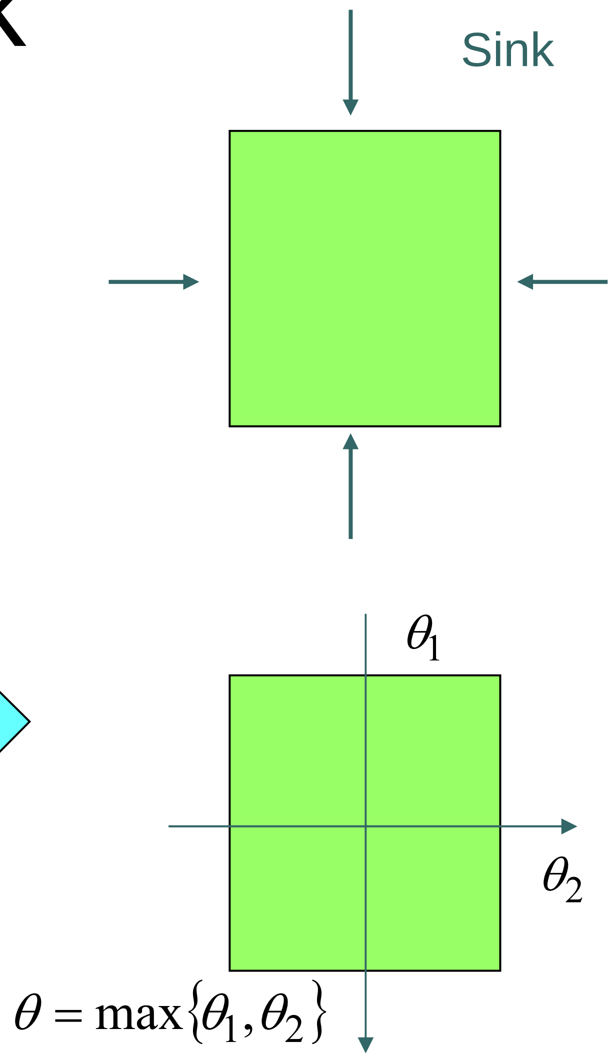
# Modeling of mesh



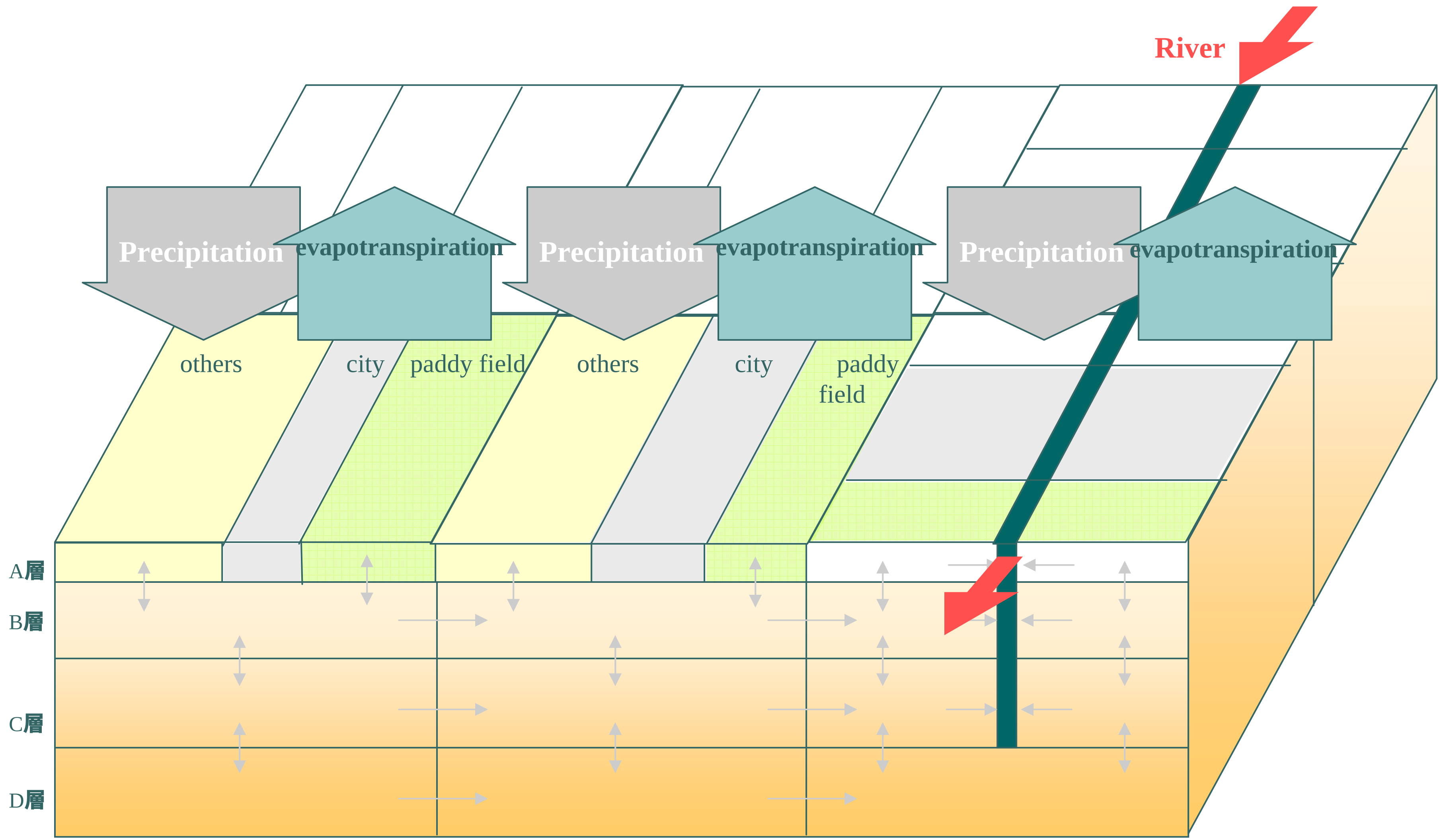
# Channel network



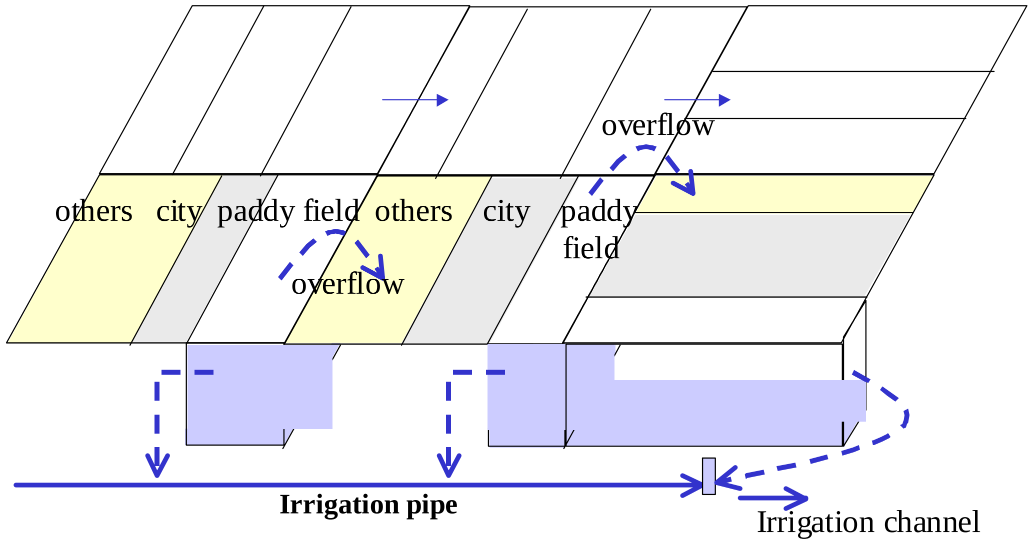



# River basin structure with meshes

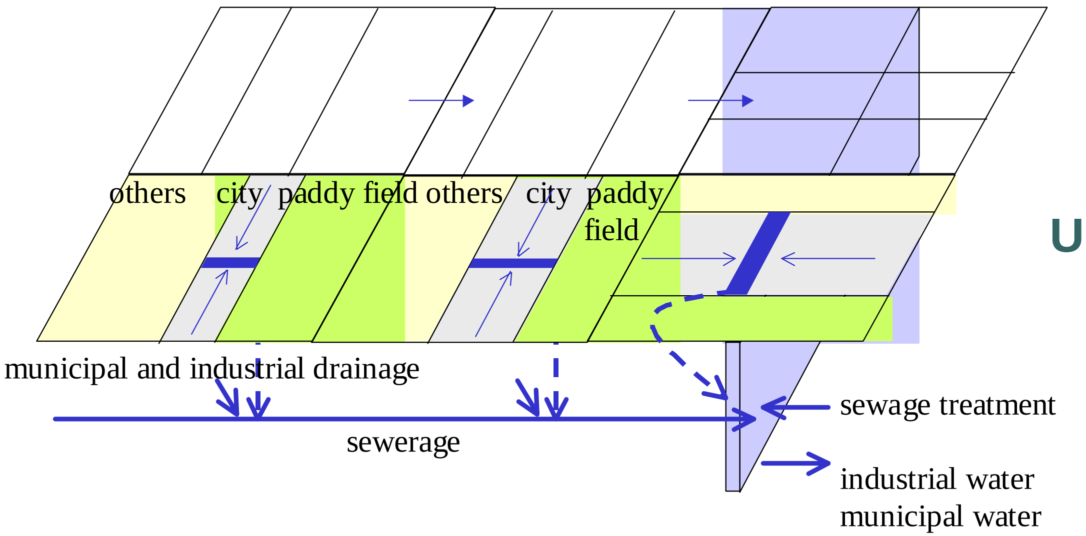


# Model components

## Paddy field



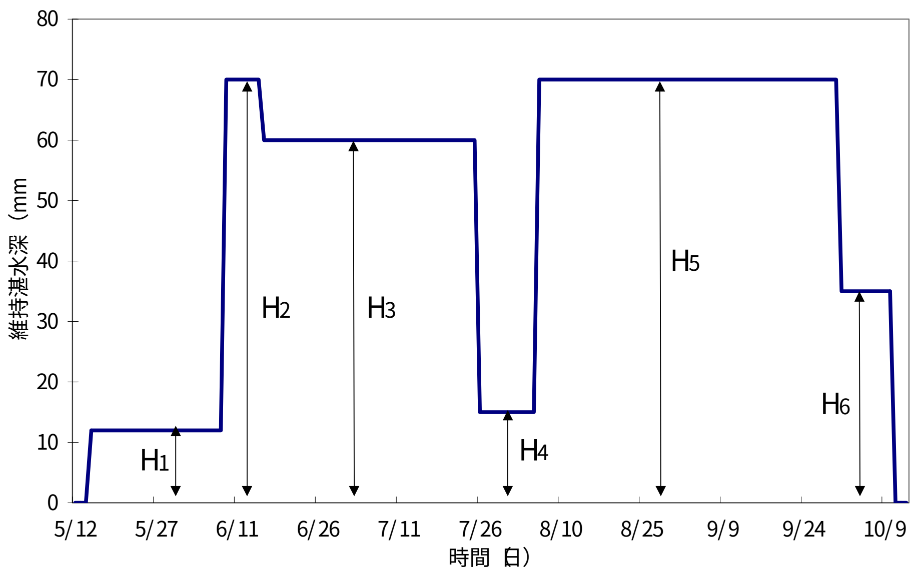

## Urban drainage



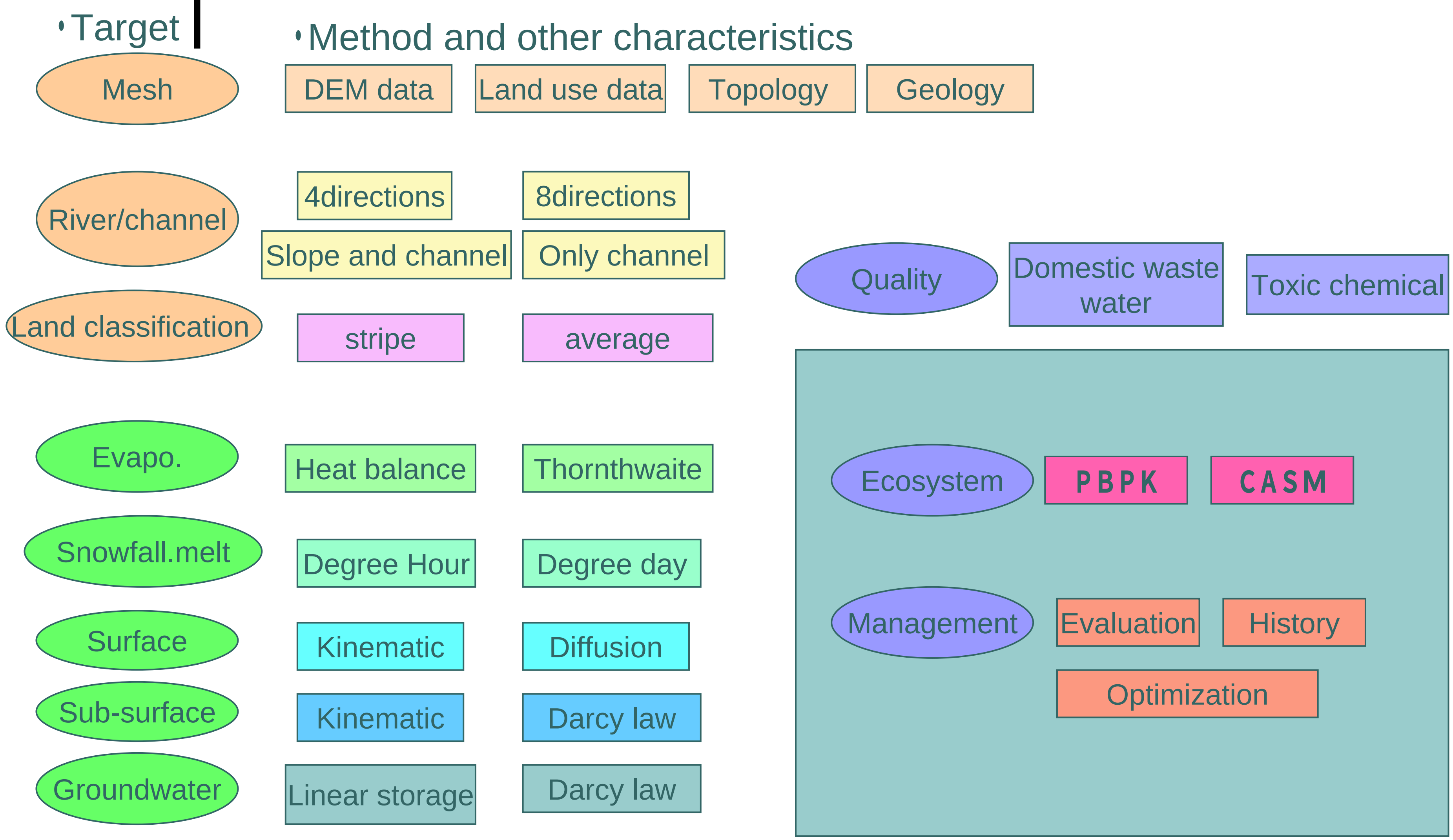
# Runoff from Paddy Field

Evapotranspiration: Heat Balance Method

Runoff: Tank Model Method for irrigation period

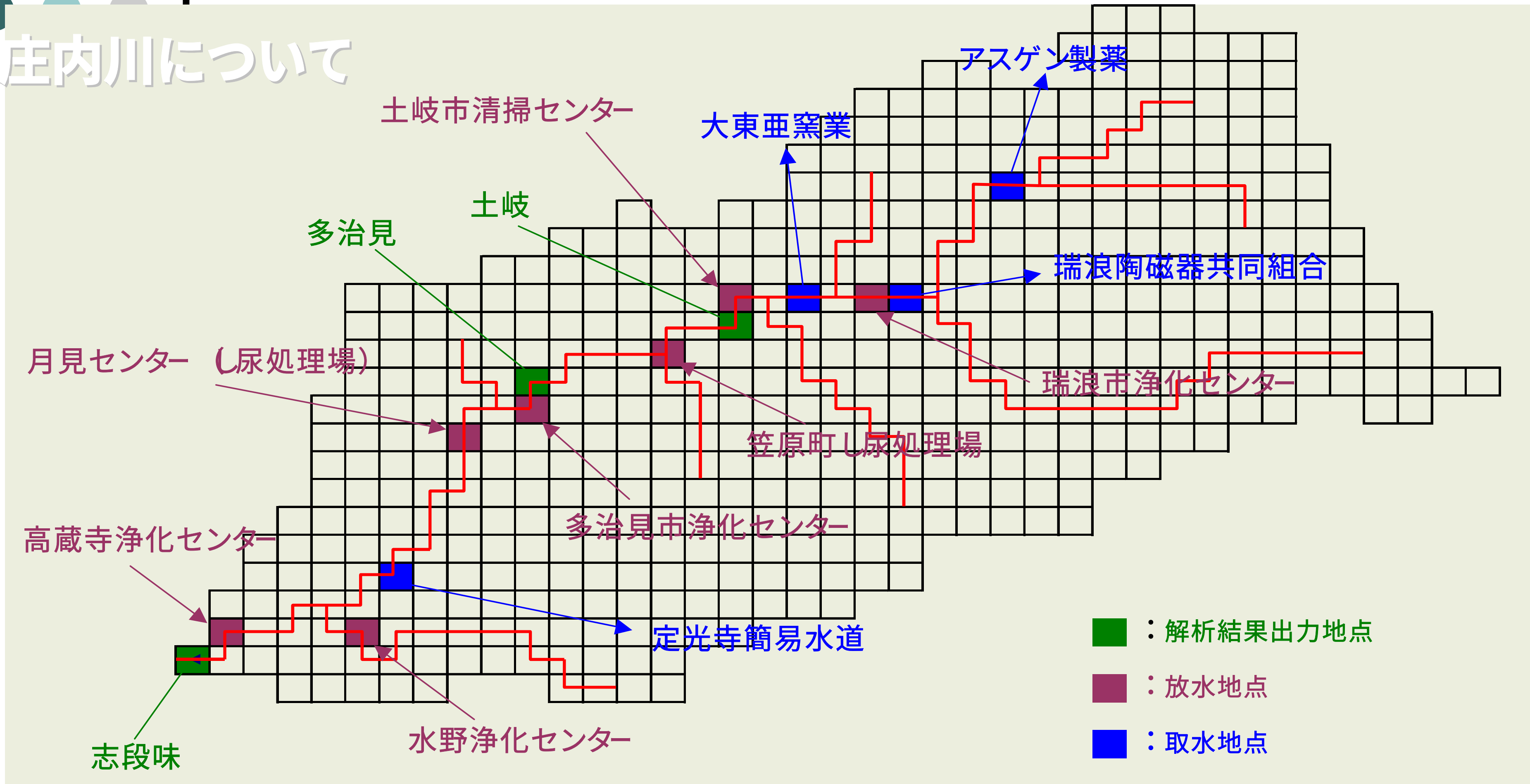
# Selection of components



# Application of distributed runoff model

- Water quantity in the real river
- Water temperature
- Water quality on domestic pollution
- Ecosystem affected with endocrine disrupter

# I. Water quantity in the real river

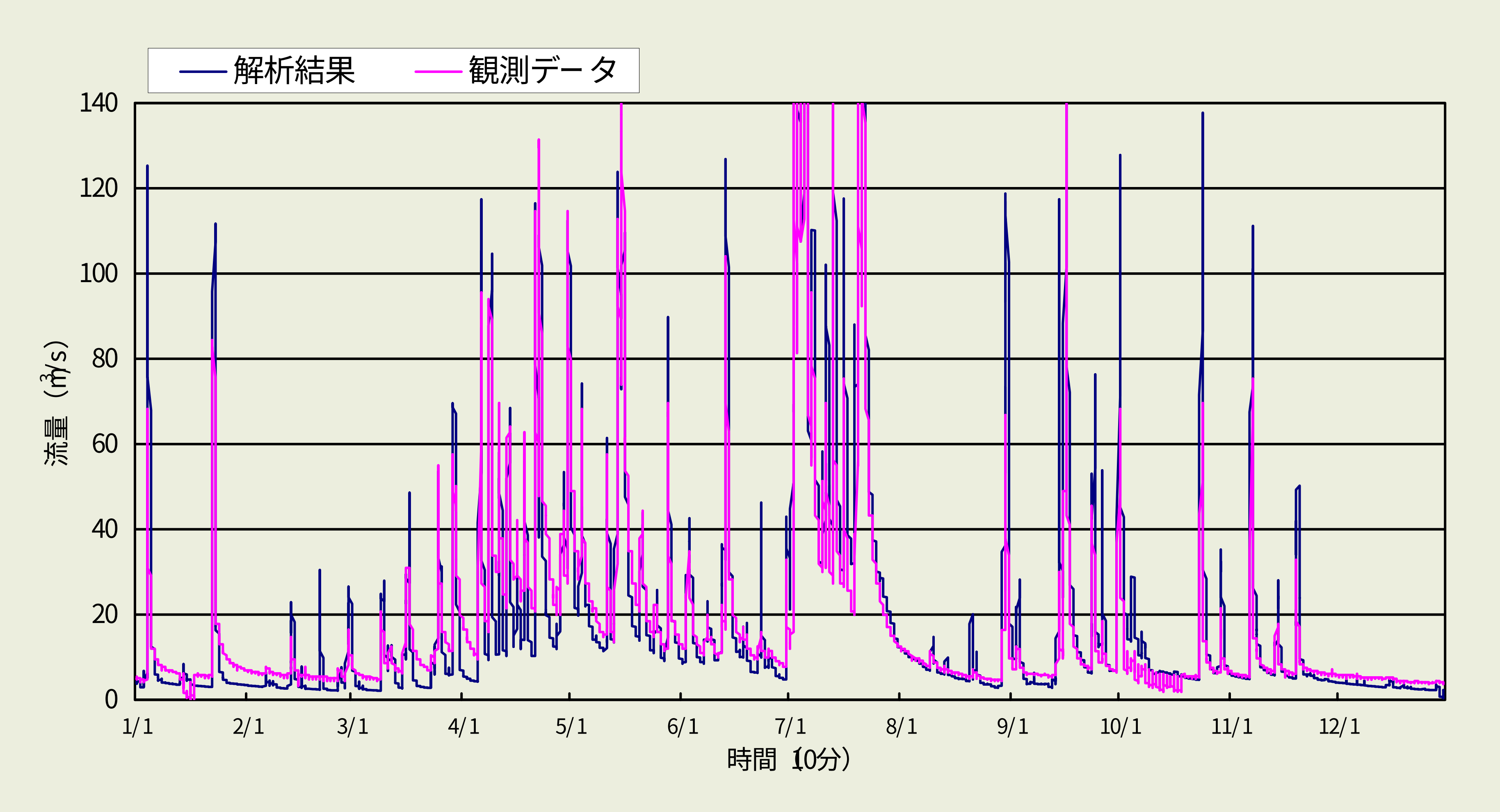



Mesh scale：1km　　Total mesh number：504

Analytic period：1995.1.1日〜12.31

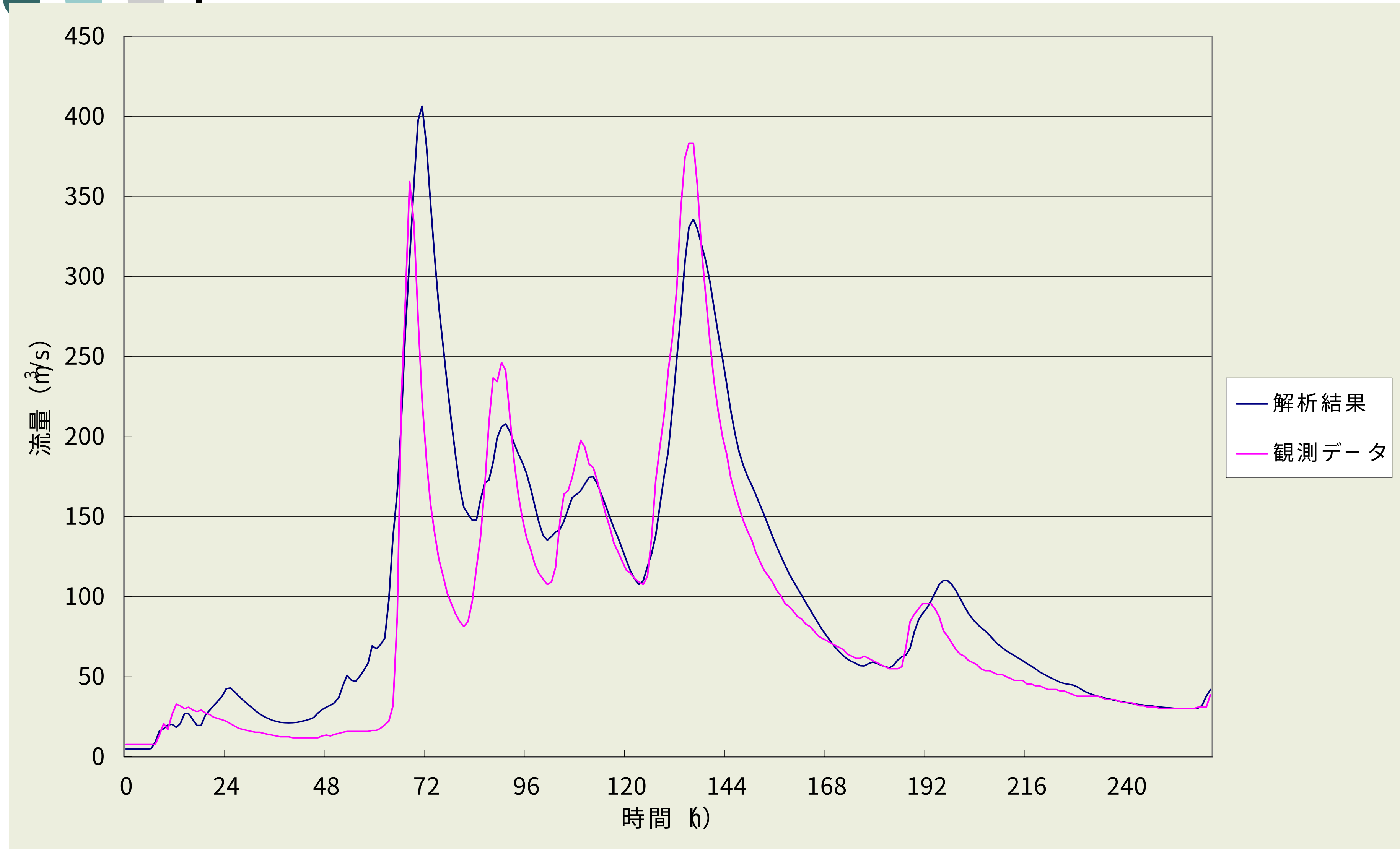
Rainfall observation：8 points

# Simulated results of long-term hydrographs

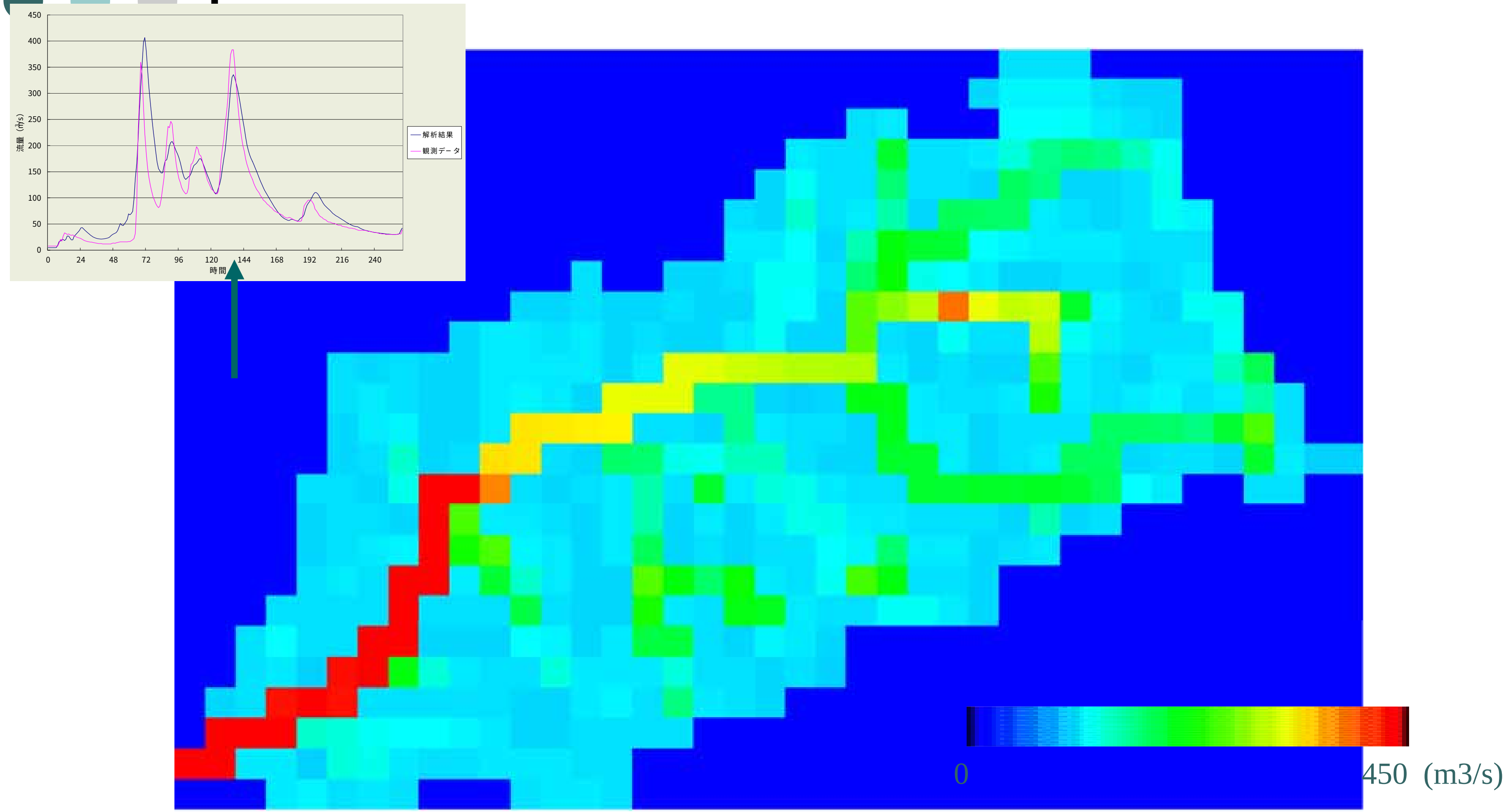



In 1995 at Shidami (lowest point)

# Simulated results of flood hydrograph

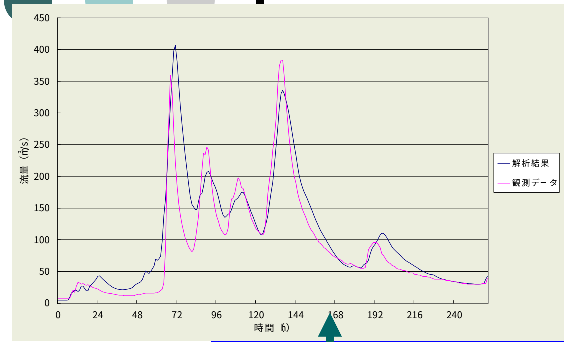


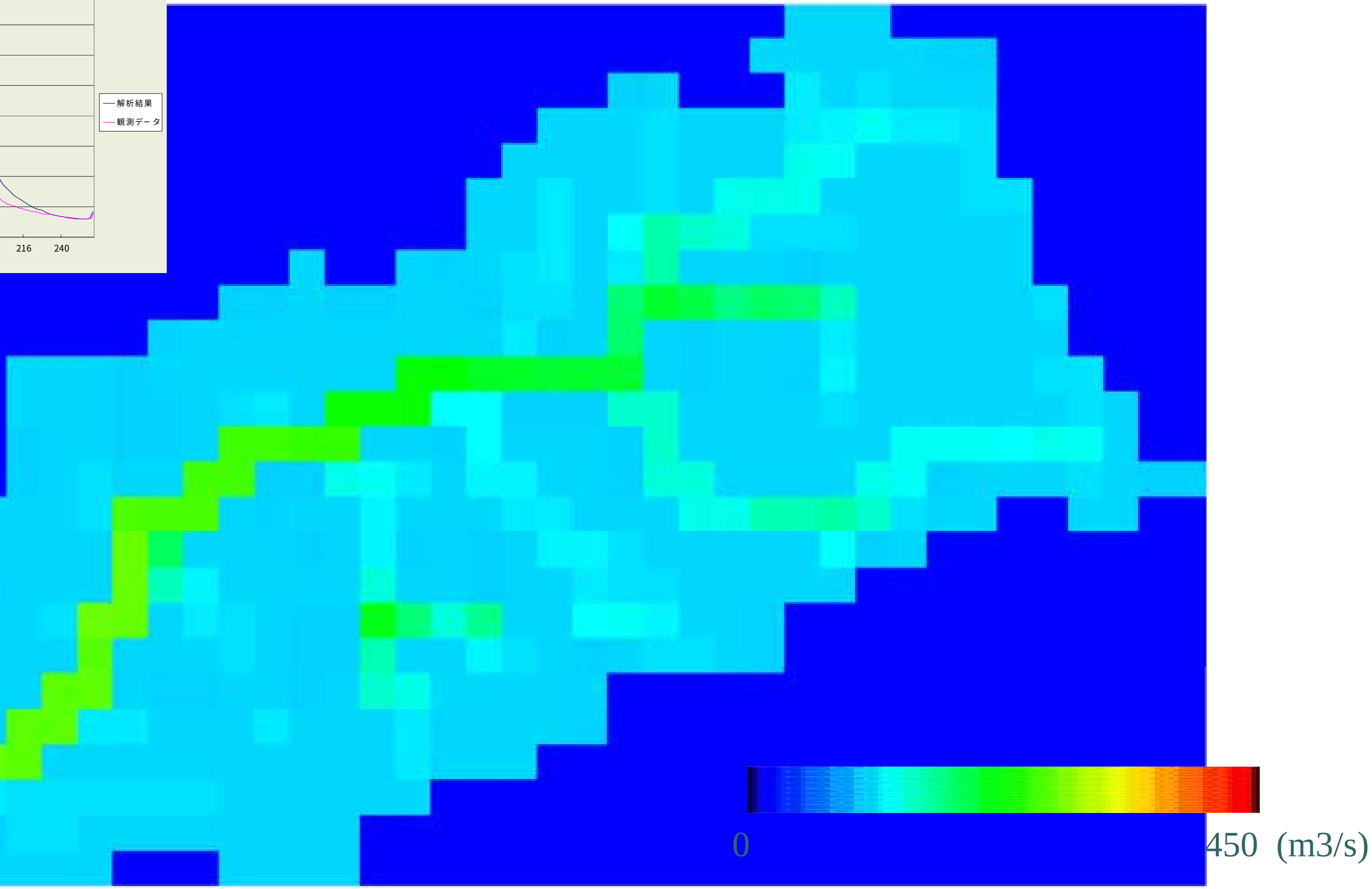
# Time 23



450
400
350
300
250
200
150
100
50
0
流量（㎥/s）
0
24
48
72
96
120
144
168
192
216
240
時間
解析結果
観測データ
0
450 (m3/s)

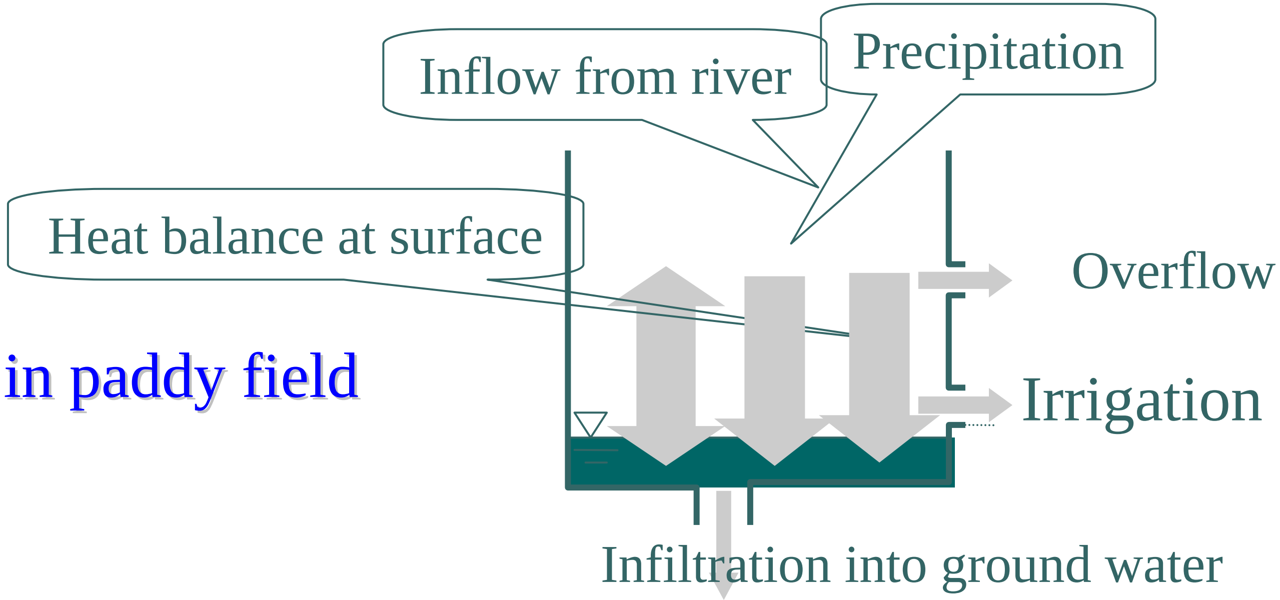
# Time 29


流量 (㎥/s)
450
400
350
300
250
200
150
100
50
0
0 24 48 72 96 120 144 168 192 216 240
時間 (h)
解析結果
観測データ

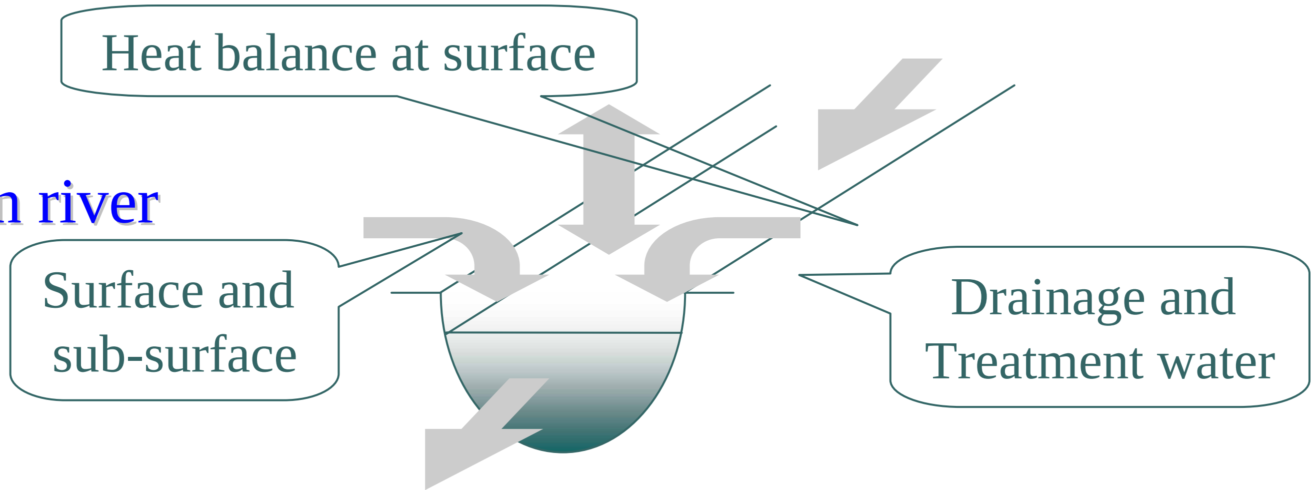

0
450 (m3/s)

# II. Water Temperature

Heat balance in paddy field




Heat balance in river



# Formulation of Water Temperature

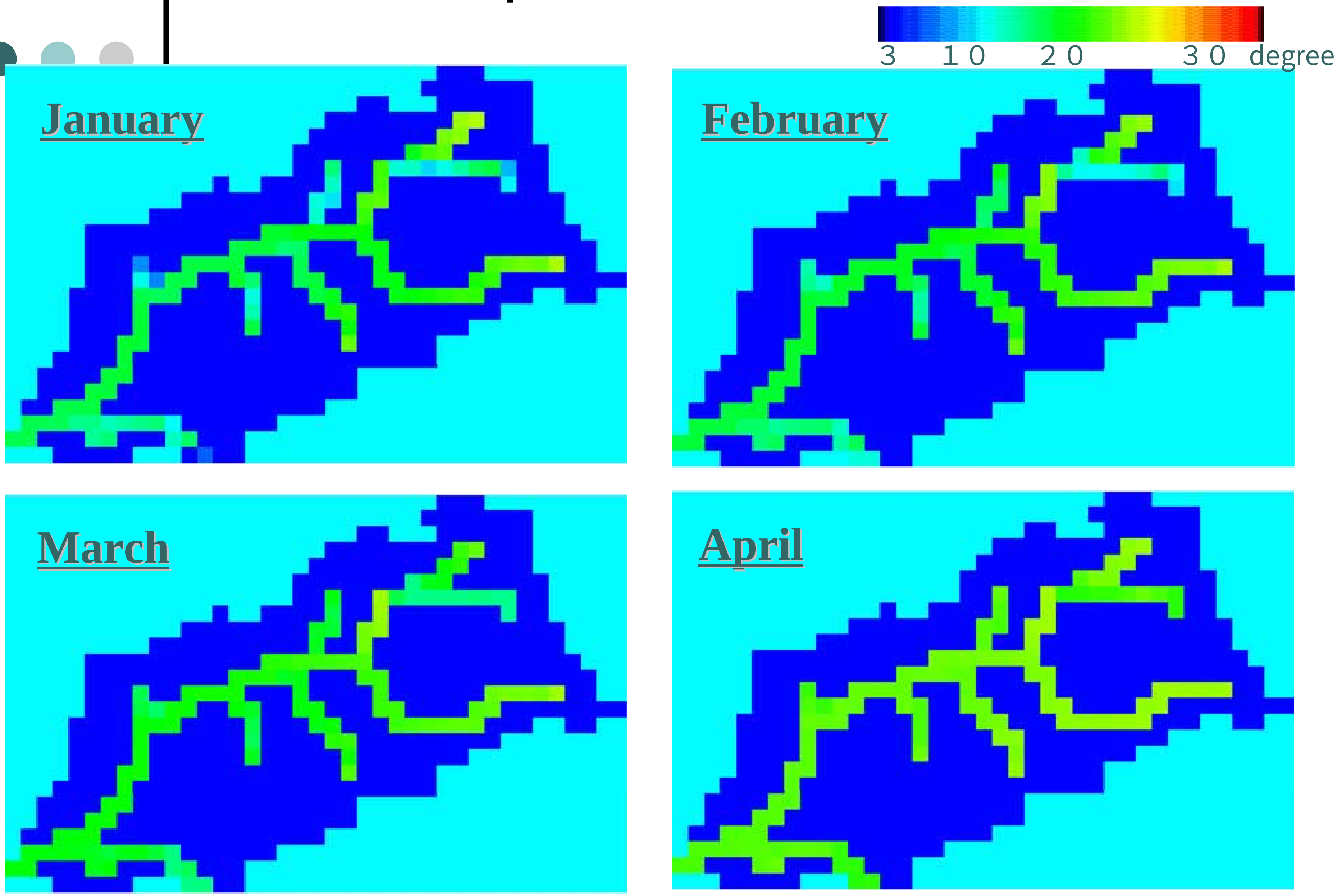
- Differential equation on heat balance

$$C\rho DY\left(\frac{\partial \theta_{riv}}{\partial \theta}\right) = H_0 + \frac{C\rho}{AW}\sum_{v} q_{Iv}(\theta_{Iv} - \theta_{riv})$$

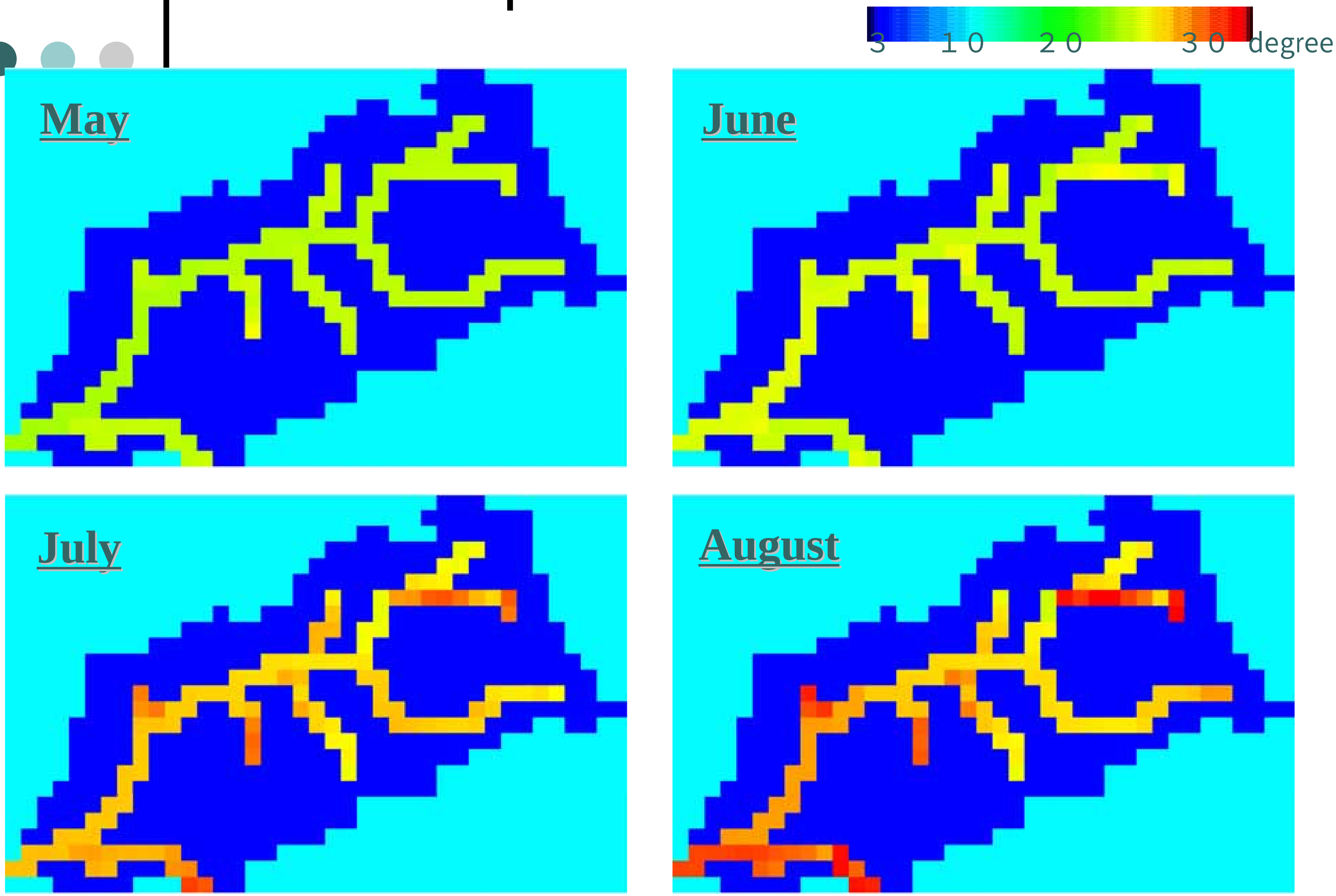
Soil temperature

$$\theta_g(y,t) = \theta_0 + De^{-y\sqrt{\pi/\chi^T}}\sin\left(\frac{2\pi}{T}t - y\sqrt{\frac{\pi}{\chi^T}}\right)$$

# Water Temperature in the River



# Water Temperature in the River



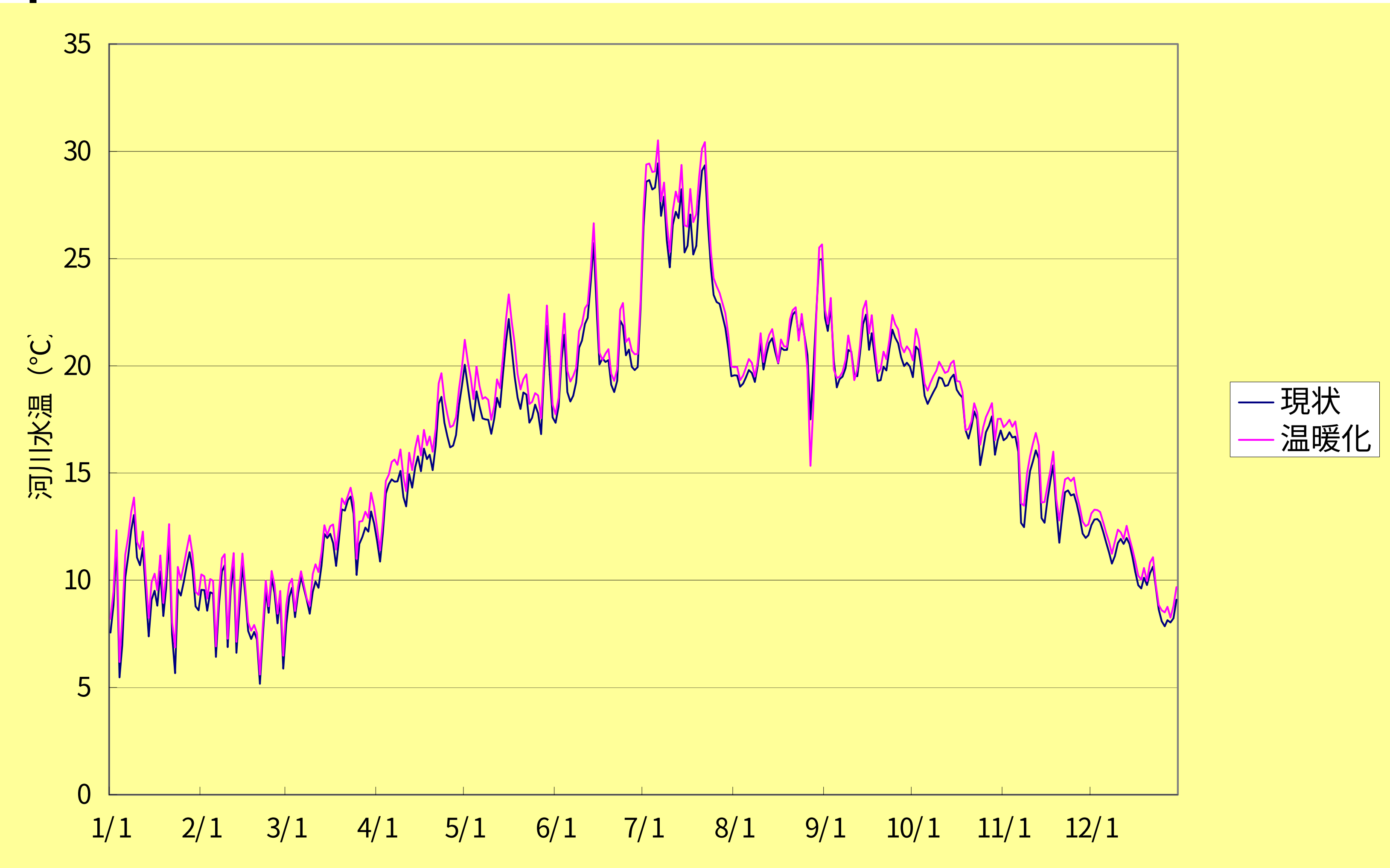
# For Climate Change

**Temperature Rise of 2℃**

**Affected Factors :**

- Evapotranspiration
- Ground surface temperature
- Precipitation temperature
- Heat balance at water surface

# Analyzed Results on Water Temperature in the River




**Averaged temperature rise ： 0.61℃**

# Impact analysis of climate change

- Present average（°C）Climate change（°C）Changed volume（°C）

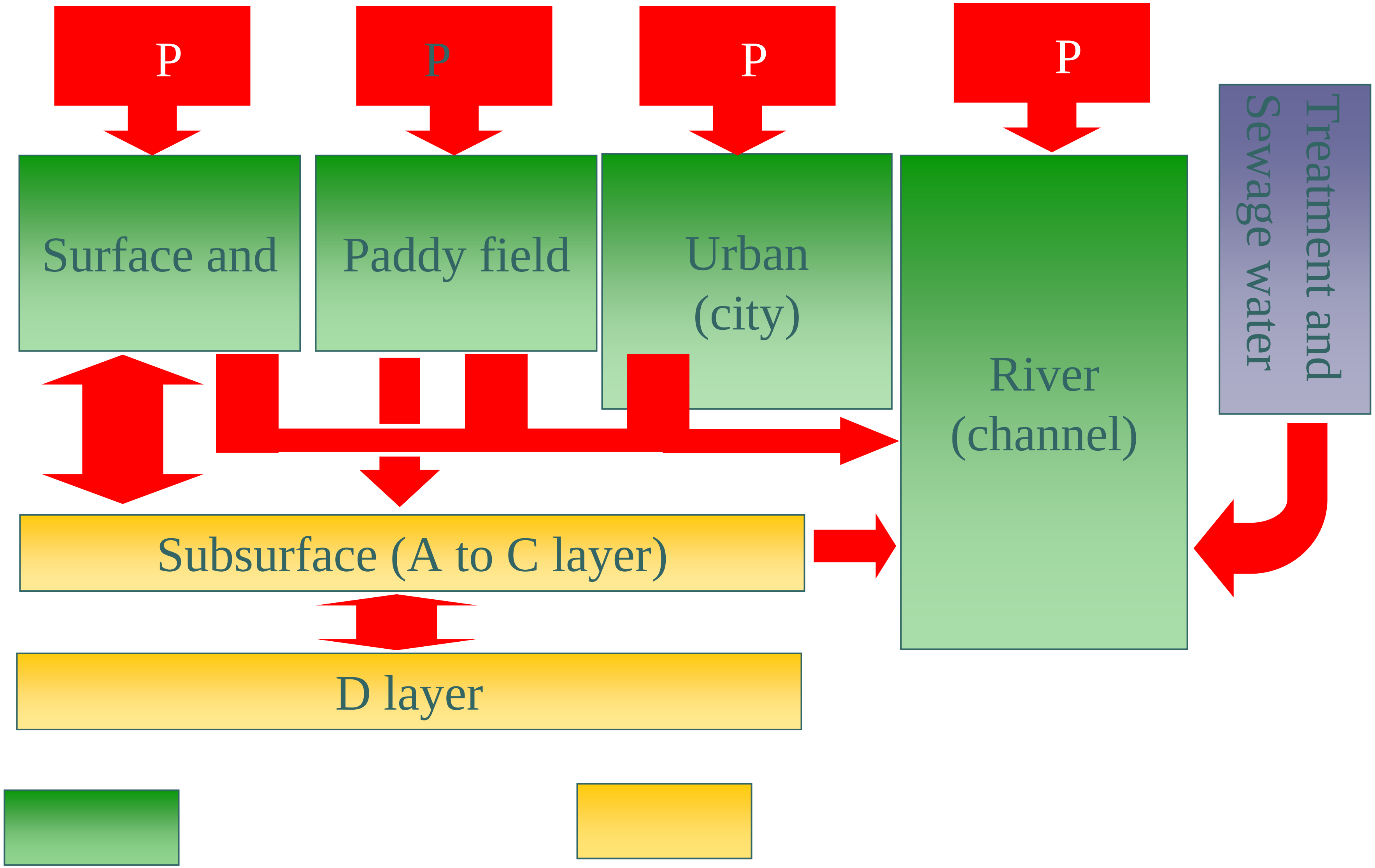
Surface 17.92 19.30 +1.32

Precipitation 11.66 12.52 +0.85

Paddy field 22.02 19.95 -2.07

River discharge

# III. Water Pollution



# Formulation of Water Quality

In the river channel:

$$A\frac{dCiSi}{dt} = Cnp\ RS\ A - k1CiSiA + k2PnpA + Li\,in - Li\,out$$

In the groundwater:

$$A\frac{dCiSi}{dt} = Li\,in - Li\,out - ki\left(CiSi - r\frac{Pi + Pio}{Pio}PiSiMAX\right)A$$

*Si*:storage volume at *i-layer* (m), *Ci*: concentration, *Cnp*:concentration in precipitation (mg/m$^3$), Pi:piling load of pollution（mg/m$^2$), *SiMAX*: maximum storage volume at *i-layer* (m), *k1* (1/h), *k2*, (1/h), *kdnp* (1/h), *ki* (1/h), *kdi*: parameters on absorption and dossolution

# Emission of water quality

House

$$PLH = unit\ requirement\ from\ living \times emission\ rate\ in\ a\ house \times emission\ rate\ of\ considered\ pollution \times regular\ population\ in\ a\ mesh$$

Factory

$$PLI = unit\ requirement\ of\ waste\ water\ from\ factory \times shipping\ amount\ of\ products \times emission\ rate\ of\ considered\ pollutant\ in\ a\ mash$$

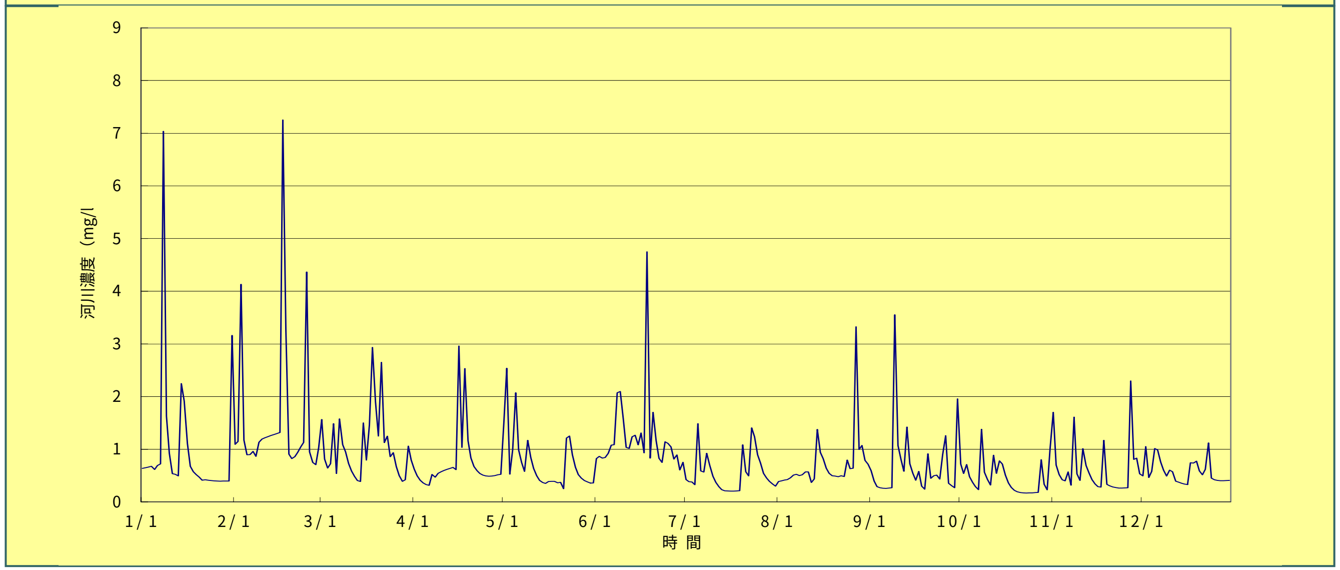
Company

$$PLC = unit\ requirement\ of\ waste\ water\ from\ living \times emission\ rate\ out\ of\ house \times emission\ rate\ of\ considered\ pollution \times pullution\ of\ considered\ job\ in\ a\ mesh \times rate\ of\ working$$

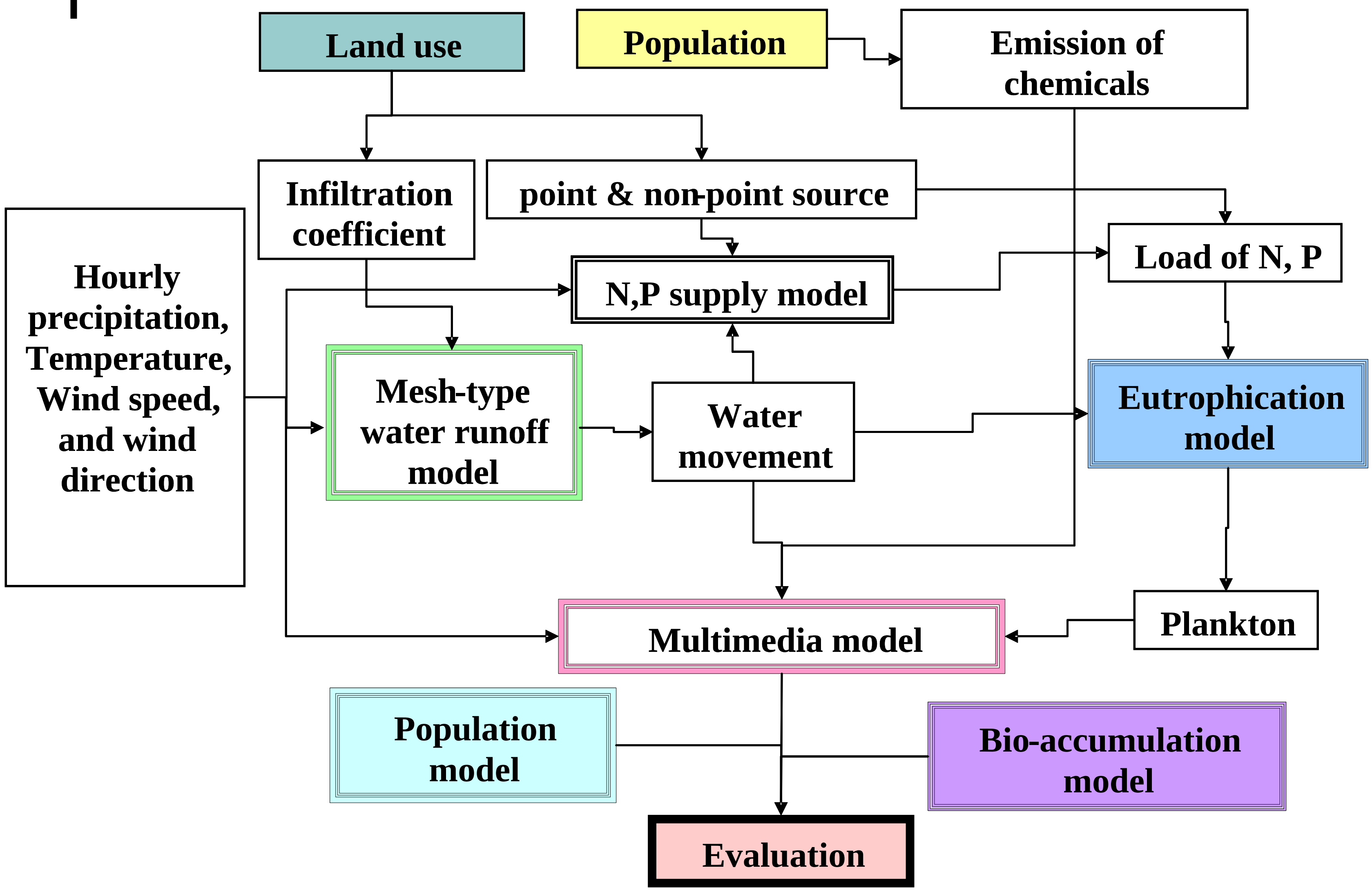
# Concentration of T-N and BOD in the River

T-N


BOD

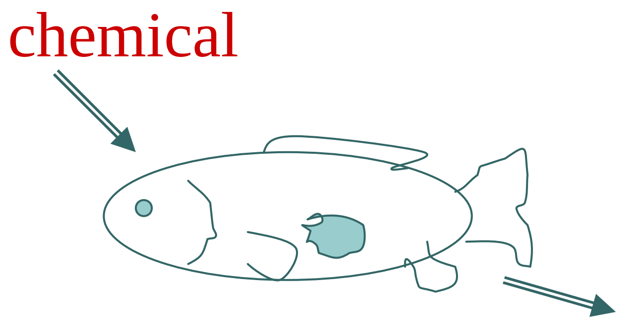


# IV. Water circulation processes for ecosystem



# Formulation of ecosystem

Adoption through gills,
Accumulation in the body of fish,
Dilution with excretion




$$\frac{dCF(t,T)}{dt} = KF_w CF_w + \sum_{n=1}^{N} \beta\, QD_n CF_{en} - (KO + GF)CF(t,T)$$

Population of fish through exposure of chemical into fish

$$\frac{dNF}{dt} = \gamma\, NF - \frac{\gamma}{EC} NF^2 - h_f (TCF(t) - TCF0) NF$$

Impact index

$$Re = \frac{NFo - NF}{NF}$$

# Time sequence of Bentiocarb at downstream point