Unsteady Flow

Back to Reynold's Transport Theorem
Mass conservation $\beta=1$
Momentum β=velocity (vector, u,v,w)

1 relates to some measure of quantity of waters
1 relates to some measure of quantity of momentum

General $\frac{dB_{system}}{dt}=\frac{d}{dt}\int_{V}\beta\rho dV+\int_{CS}\beta\rho V\cdot dA$

Momentum $\frac{dB_{system}}{dt}=\frac{d(momentuem)}{dt}=\frac{d}{dt}\int_{V}\beta\rho dV+\int_{CS}\beta\rho V\cdot dA$

Newton's 2nd law:
Rate of change of momentum
= sum of all forces acting on system

$$\sum F_{body}+\sum F_{surface}=\frac{d}{dt}\int_{V}\underline{v}\rho dv+\int_{CS}V\rho VdA$$

Most general representation of forms include "Corrolis Force"
Corrolis Force: Related to applreability of Newton's Law in a fixed frame of reference
Non-accelerating

Coriolis Force: $\int_{cv}[\frac{d^2\underline{R}}{dt^2}+2\underline{\Omega}\times\underline{V}+\Omega\times(\Omega\times r)+\frac{d\Omega}{dt}\times r]\rho dV$

- Neglect corrolis Force
- Convert surface integrals to volume integrals(Grass thm)
- Navier-Stokes equations (3D, instantaneous)
- Apply turbulence time averaging
- Reynold's Equations(RANS)
- Average over depth
- 2D shallow water equations(faster than Average over depth)

Average over cross section

1D shallow water wave equations de ST.Venant equations

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q$$ Mass conservation "continuity"

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial y}{\partial x}+gAS_f=0$$ Momentum conservation Dynamic equation

A(x,t)=x sectional area
Q(x,t)=volumetric discharge
q(x,t)=lateral distributed inflow/outflow

Assume the density is constant

y(x,t)=watersurface elevation
Sf(x,t)="energy slope"

Q=AV V=velocity
Area(x,t) or A(y)$|_x$
y(x,t)
We can think of Area A or W.S elevation y as a dependent variable
(or depth h(x,t))
We don't see bed slope in this equation so what happen to the gravity forces?

$gA\frac{\partial y}{\partial x}$ Capture both gravity and pressure forces

Dependent variables (unknowns): (Q A) or (Q y) or (Q h) or ….
(v, y) or….
Independent variables: x=longitudinal position
t=time

Assume steady flow stress equations can be used for unsteady flow
(Chezy, Manning-Stucter, Hozen-Williams, fruction-factor equation can be used in unsteady flow)

Manning-struchter: $V=k_{str}R^{2/3}S_f^{1/2}=1/nR^{2/3}S_f^{1/2}=1/nR^{2/3}S_f^{1/2}$

SI UK

A=A(x,t)

$$Q = A_{str}\frac{A^{5/3}S_f^{1/2}}{p^{2/3}}S_f^{1/2}$$

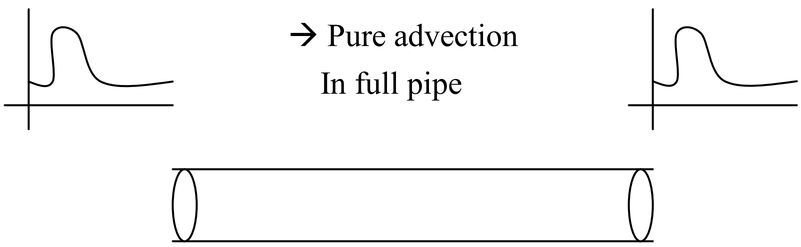
K(x,t)=”conveyance”

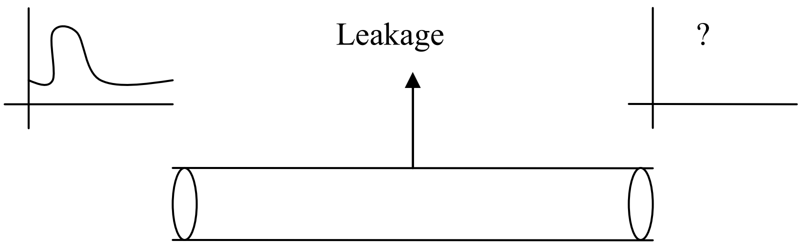
So $S_f = \frac{Q^2}{k^2} = \frac{Q|Q|}{k^2} = S_f$

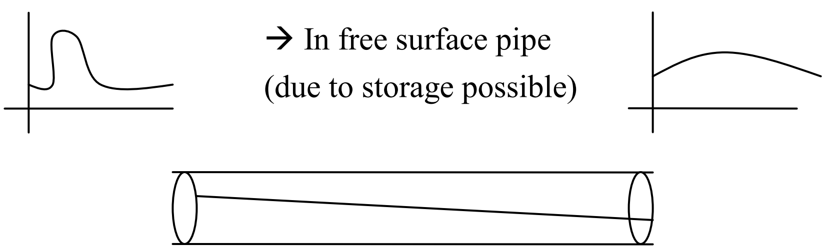
Note conveynce can also be from “Chezy equation, Darcy-weis brach, Hagin, Hazer-williams etc.

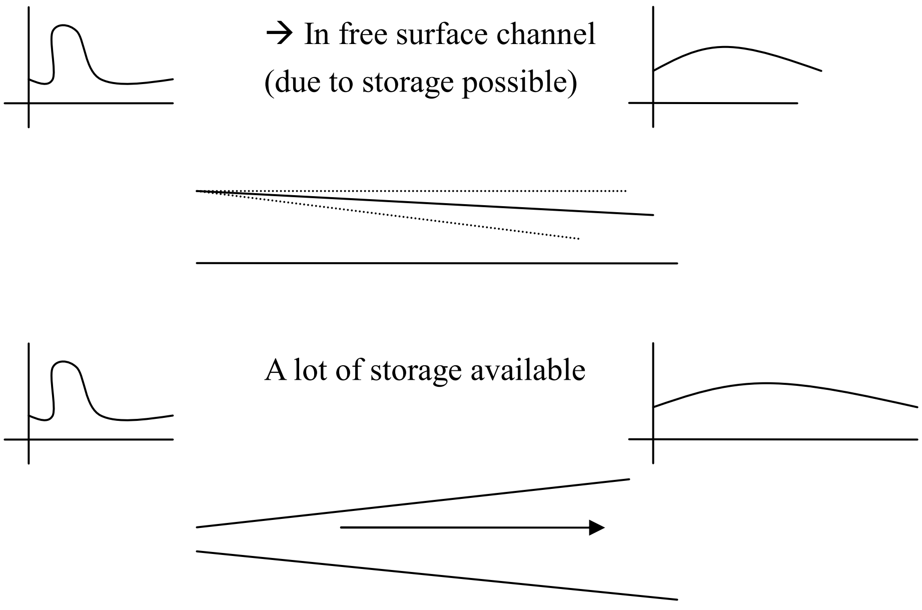
K(x,t) is a parameter for any cross section at any time

Unsteady flow has both hyperbolic (advection-like) and parabolic (diffusion-like) properties.

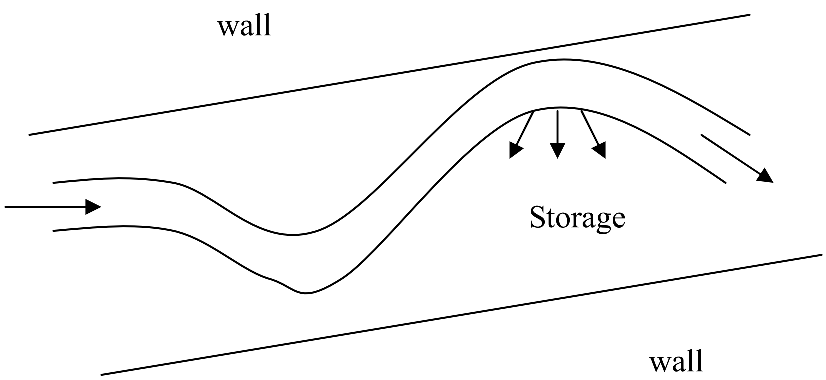








Flood wave “attenne tron”
(Spreading out in space, decrease in peak discharge)
Is caused by nze +fall of W.S elevation (Water being stored and unstored) and changes in width of W.S)



y(t)results from Q(t) to do something

Numerical methods for de st. vanant equation

- Finite element: used by some – not natural no particular advantage in one dimension
- Method of characteristics: very accurate for rapidly flow Natural for hydraulic transients. Award for flood computations
- Finite difference methods: most common

- Explicit method are usually conditionally stable but easy
- Implicit method are usually unconditionally stable by require solution of systems of equations
  (Mike11 Abbott-Ionercou Scheme)

Courant like number = $\dfrac{(u \pm c)\Delta t}{\Delta x}$, $c \approx \sqrt{gh} = \sqrt{g\dfrac{A}{T}}$

Preismmann (1917) Noted that Richtmyer

$$\frac{\partial c}{\partial t} = \frac{\theta}{\Delta x^2}\frac{C_{i+1}^{n+1} - 2C_i^{n+!} + C_{i-1}^{n+1}}{} + \frac{(1-\theta)}{\Delta x^2}(C_{i+1}^n - 2C_i^n + C_{i-1}^n)$$

Apply to de st. Vennant equation

Use 1/2 weighting for $\partial/\partial t$

Use θ weighting for $\partial/\partial t$

If f(x,t)is one of our variables(Q,A,y,p,Sf)

$$f(x,t) \approx \theta\frac{f_i^{n+!} + f_{i+1}^{n+1}}{2} + (1-\theta)(\frac{f_i^n + f_{i+1}^n}{\Delta x})$$

$$\frac{\partial f}{\partial x} \sim$$

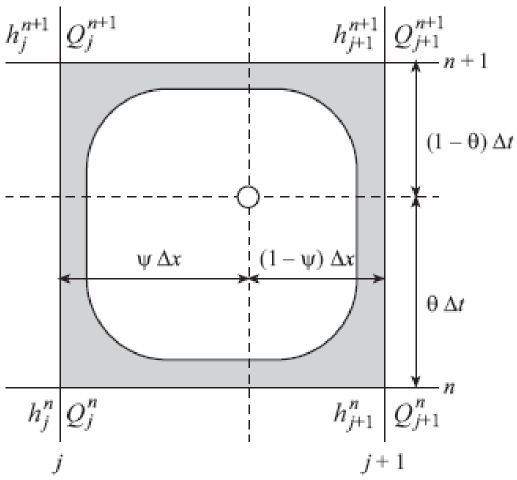
$$\frac{\partial f}{\partial t} \sim$$

Continuity equation

$$\frac{\partial h}{\partial t} + \frac{A}{b_s}\frac{\partial Q}{\partial x} = 0$$

$$b_s \frac{\partial h}{\partial t} \sim \psi \, b_{s\,j+1}^{\,n+\frac{1}{2}} \left(\frac{h_{j+n}^{n+1} - h_{j+n}^{n}}{\Delta t} \right) + (1-\psi) \, b_{s\,j}^{\,n+\frac{1}{2}} \left(\frac{h_j^{n+1} - h_j^{n}}{\Delta t} \right)$$

$$\frac{\partial Q}{\partial x} \sim \theta \left(\frac{Q_{j+n}^{n+1} - Q_j^{n+1}}{\Delta x} \right) + (1-\theta) \left(\frac{Q_{j+n}^{n} - Q_j^{n}}{\Delta x} \right)$$



=> $$A1_j \, h_j^{n+1} + B1_j \, Q_j^{n+1} + C1_j \, h_{j+1}^{n+1} + D1_j \, Q_{j+1}^{n+1} = E1_j$$

1 equation 4 unknowns

When we do this for the dynamic equation we will get one nonlinear algebra equation in same 4 unknowns

Let s use the linear continuity equation to see how we can procees

Taylor series expansion: $F_1^{n+1} = F_1^{n} + \Delta F_1$

Previous value !=0

correction to previous value of F1(unknown)

Taylor: $\Delta F_1 = \frac{\partial F_1}{\partial A_1} | \Delta A_i + \frac{\partial F_1}{\partial A_{i+1}} + \frac{\partial F_1}{\partial Q_i} \Delta Q_i + \frac{\partial F_1}{\partial Q1_{i+1}} \Delta Q_{i+!} \ldots$ Newton-Paphson assumes

these can be neglected

$F_1^{n+1} = F_1(4unknown) + \Delta F_1$

Estimate at iteration level m (known) !=0 unless the A Q estimates are perfect

→(Known)

We want to F to be zero

Find corrections ΔA_{i+1}^{n+1} etc. that force F =0

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Preismann's Finite difference method-1960

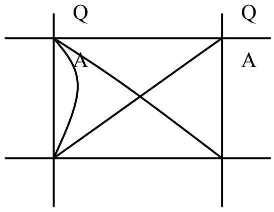
"Implicit 4-point scheme"

"Amien and Wang"

Unconditionally stable for θ >=0.5 we use 0.55

θ =1.0 give most smoothing

Abbott' s book



Continuity Equation: $\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0$

Assume no continuous lateral (Inflow)

$$\frac{1}{2\Delta t}\left(A_i^{n+1}-A_i^n\right)+\frac{1}{2\Delta t}\left(A_{i+1}^{n+1}-A_{i+1}^n\right)+\frac{\theta}{\Delta x}f(Q)+\frac{(1-\theta)}{\Delta x}f(Q)=0$$

When we do this for dynamic (momentum) equation, we obtain anther function

Function is Nasty, complicated, non linear algebra equations

Newton-Raphson iteration will be used to solve the non linear system of equations

Consider **Scalar** example:

For totally non linear equation

Taylor Series:

$$f(x^{n+1})=f(x^n)+\frac{df}{dx}|_{x^m}\ \Delta x+.....$$

Ignore all but 1st term

$$f(x^{n+1})=f(x^n)+\frac{df}{dx}|_{x^m}\left(x^{n+1}-x^n\right)+.....=0$$

=> $x^{n+1}-x^n=-\frac{f(x^n)}{\frac{df}{dx}|_{x^n}}$

$$x^{n+1} = x^n - \frac{f(x^n)}{\frac{df}{dx}|_{x^n}}$$

m= 0,1,2,3… until m is acceptably small

Back to algebra discretised continuity equation

$$F_1(A_i^{n+1}, A_{i+1}^{n+1}, Q_i^{n+1}, Q_{i+1}^{n+1}) = 0$$

$$F_1({}^{m+1}A_i^{n+1}, {}^{m+1}A_{i+1}^{n+1}, {}^{m+1}Q_i^{n+1}, {}^{m+1}Q_{i+1}^{n+1}) = F_1({}^{m}A_i^{n+1}, A_{i+1,}Q_i, Q_{i+1}) + ...$$

$$\frac{\partial F_1}{\partial A_i}\bigg|_m \Delta A_i + \frac{\partial F_1}{\partial A_{i+1}}\bigg|_m \Delta A_{i+1} + \frac{\partial F_1}{\partial Q_i}\bigg|_m \Delta Q_i + \frac{\partial F_1}{\partial Q_{i+1}}\bigg|_m \Delta Q_{i+1}$$

Note that first estimations m=0 are the values at end of last time step

$${}^0A_i^{n+1} = A_i^n etc$$

Note that $\Delta A = b\Delta y$

It will be convenient to use Q and y as dependence valuables in F2

Our continuity equations becomes for 1st iteration:

$$A_1\Delta y_{i+1} + B_1\Delta Q_{i+1} = C_1\Delta y_i + D_1\Delta Q_i + G_1$$

A1=$\frac{b_{i+!}}{2\Delta t}$ Known from $\frac{\partial F_1}{\partial y_{i+!}}$

B1=D1=$\frac{\theta}{\Delta x}$ $\frac{\partial F_1}{\partial Q_{i+1}}$

C1=$\frac{-b_i}{2\Delta t}$

G1=$\frac{1}{2\Delta t}(A_i^n - {}^mA^{n+1} + A_{i+1}^n - {}^mA_{i+1}^{n+1}) + \frac{\theta}{\Delta x}({}^mQ_{i+1}^{n+1} - {}^mQ_i^{n+1}) + \frac{1-\theta}{\Delta x}(Q_{i+1}^n - Q_i^n)$

All of these are known at sort of iteration

Note that in steady flow, $\Delta A = \Delta Q = 0$

=>F1=0 would just be G1=0

=>$\frac{\theta}{\Delta x}({}^mQ_{i+1}^{n+1} - {}^mQ_i^{n+1}) + \frac{1-\theta}{\Delta x}(Q_{i+1}^n - Q_i^n) = 0$=>$Q_{i+1} = Q_i$ insteady flow

Same procedure for dynamic equation F2=0

$$A_2\Delta y_{i+1} + B_2\Delta Q_{i+1} = C_2\Delta y_i + D_2\Delta Q_i + G_2$$

→Ugly coefficient known

Why we need iterate?
Usually not necessary, 1st NR iteration is very accurate
It can be necessary when width changes rapidly of four full pipes flow

Full Pipe flow "Presimann slot"

How do we deal with Area, width, conveyance, function for a cross section
If we have a simple section
Easier to use the equation for the geometry
(for every point i, in every iteration, in every time step, our de St. Vanat equations require evaluation of A P K)
If we have natural cross sections equation

- Some people fit a polynomial to channel slope
- Some tabular A, b, p, k for rapid access during calculation.

Apply our discretised St Vanannt eq

$A_1 \Delta y_{i+1} + B_1 \Delta Q_{i+1} = C_1 \Delta y_i + D_1 \Delta Q_i + G_1$ => F1=0

$A_2 \Delta y_{i+1} + B_2 \Delta Q_{i+1} = C_2 \Delta y_i + D_2 \Delta Q_i + G_2$ =>F2=0

2 equations
4 unknowns
If our channel has i-1 computational regions * 2 equations = 2I-2 equations available
If our channel has 2 I unknowns
So need 2 more equations= 2 boundary conditions

We have a system of 2I linear algebraic equations in 2I unknowns
To solve in each iteration of each time step

$$[A]\begin{vmatrix}\Delta y \\ \Delta Q \\ \Delta y \\ \Delta Q\end{vmatrix}\{=[a1]$$

Solution is [a1]*[A]$^{-1}$

Double sweep solution of the system sets the stage for the treatment of networks
We assume $\Delta Q_i = E_i \Delta y_i + F_i$
Eliminate ΔQ_i from F1=0 F2=0 equations
=> $\Delta y_i = L_i \Delta y_{i+1} + M_i \Delta Q_{i+1} + N_i$
L, M, N to determined

$$L_i = \frac{A_1 D_2 - A_2 D_1}{(C_1 D_2 - C_2 D_1)}$$

$$M_i = \frac{B_1 D_2 - B_2 D_1}{(C_1 D_2 - C_2 D_1)}$$

$$N_i = \frac{D_1 G_2 - D_2 G_1}{(C_1 D_2 - C_2 D_1)}$$

From cross multiplying F1=0, F2=0 then substrcting

Now we go back to F1=0, or $A_1 \Delta y_{i+1} + B_1 \Delta Q_{i+1} = C_1 \Delta y_i + D_1 \Delta Q_i + G_1$

$$\Delta Q_{i+1} = E_{i+1} \Delta y_{i+1} + F_{i+1}$$

$$E_{i+1} = \frac{L_i (C_1 + D_1 E_1) - A_1}{B_1 - M_i (C_1 + D_i E_i)}$$

$$F_{i+1} = \frac{N_i (C_1 + D_1 E_1) + D_1 F_1 + G_1}{B_1 - M_i (C_1 + D_i E_i)}$$

Recursion relationships for a forward sweep

For a single channel where a possible solutiomn structure

Loop on time steps n=0,… n

  Apply a BC to initialize the E1 F1

  Loop on iterations m=0….?

    Loop on computational regions, i=1….I-1

      Acquire geometric properties of cross section at I, i+1

      Compute A1,B1,C1,D1,G1

      Compute L,M,N for each and store

      Compute Ei+1and Fi+1

We have $\Delta Q_{i+1} = E_{i+1} \Delta y_{i+1} + F_{i+1}$

Apply 2nd BC eq

Solve 2 equations in 2 unknowns to get $\Delta Q_I \Delta y_I$

Loop on points, I-1,…..1

  Use Δy_{i-1}=….

  ΔQ_{i-1}=….

Are Δy and ΔQ small enough to halt iterations?

1 D unsteady flow- Preismann's Method

- Applied discretisation => 2 nonlinear algebraic egns for each computaional reach
- Apply Newton-Raphson iteration algotithem to get 2 linear algebratic equations in 4 unknowns for each computational reach.
  $A_1\Delta y_{i+1} + B_1\Delta Q_{i+1} = C_1\Delta y_i + D_1\Delta Q_i + G_1$
  $A_2\Delta y_{i+1} + B_2\Delta Q_{i+1} = C_2\Delta y_i + D_2\Delta Q_i + G_2$
- Develop the algebra relationships for double sweep solution of the algebra system (matrix inversion)
  $\Delta Q_{i+1} = E_{i+1}\Delta y_{i+1} + F_{i+1}$
  $\Delta y_{i+1} = L_{i+1}\Delta y_{i+1} + M_{i+1}\Delta Q_{i+1} + N_{i+1}$
- Now we need boundary conditions to compute our linear systems
  I=# of pts on a channel
  2I-2 linear equations
  2I unknowns ΔQ_{i+1} Δy_{i+1}

  Need 2 b.c
  $\Delta Q_1 = E_1\Delta y_1 + F_1$
  $\alpha\Delta y_1 + \beta\Delta Q_1 = \gamma$, a linealized b.c
  Solve 2 equations in 2 unknowns
  $E_1 = -\frac{\alpha}{\beta}$, $F_1 = \frac{\gamma}{\beta}$

  2 equations
  2 unknowns ΔQ_{i+1} Δy_{i+1}
  $\Delta Q_{i+1} = E_{i+1}\Delta y_{i+1} + F_{i+1}$
  $\alpha\Delta y_{i+1} + \beta\Delta Q_{i+1} = \gamma$
  Solve to obtain ΔQ_{i+1} Δy_{i+1}

- What if b.c is imposed **discharge Q(t)**

  We want ${}^{m+1}Q_1^{n+1} = Q(t_{n+1})$

  ${}^{m}Q_1^{n+1} + \Delta Q_1 = Q(t_{n+1})$

  After 1st iteration, this is ~0 but not equal to 0 due to the machine runoff

  $\alpha = 0$, $\beta = 1$, $\gamma = Q(t_{n+1}) - {}^{m}Q_1^{n+1}$

  Known from specified const

  Known from last iteration to short our

time step m=0 and $^{0}Q^{n+1} = Q^{n}$

- What if b.c is imposed **water surface elevation y(t)**?

  $$^{m+1}y_1^{n+1} = y(t_{n+1})$$

  $$^{m}y_1^{n+1} + \Delta y_1 = y(t_{n+1})$$

  $$\alpha = 0, \quad \beta = 1, \gamma = y(t_{n+1}) - {}^{m}y_1^{n+1}$$

  If $\beta = 0$ must either:

  - Cheat (by setting eq E1=…)
  - Change the double sweep relations in first reach

    $$\Delta y_1 = e\Delta Q_1 + f$$
    $$\Delta Q_1 = l\Delta y_{i+1} + m\Delta Q_i + n_i$$

- **Rating curve** at a boundary Q(y) or Q=f(y)

  We want boundary condition to be satisfied at the end of time step

  $$^{m+1}Q_{bc}^{n+1} = f\left(^{m+1}y_{bc}^{n+1}\right)$$

  Function that represents rating

  curve= $f\left(^{m+1}y_{bc}^{n+1}\right) + \frac{\partial f}{\partial y}|_m \Delta y + ...$

  Known known assume=0

  $$^{m+1}Q_{bc}^{n+1} = f\left(^{m+1}y_{bc}^{n+1}\right) + \frac{\partial f}{\partial y}|_m \Delta y + ...$$

  $$\alpha = \frac{\partial f}{\partial y}|^{m}y_{bc}^{n+1}, \quad \beta = -1, \gamma = {}^{m}Q_{bc}^{n+1} - f(^{m}y_{bc}^{n+1}) \quad (\gamma \approx 0 but \gamma \neq 0)$$

What about internal boundary conditions?

Conatability conditions

Non-flurnal conditions

Example weirs, water surface constrollers, puming stations, etc.

Make life as easy as possible:

Obtain $A_1\Delta y_{i+1} + B_1\Delta Q_{i+1} = C_1\Delta y_i + D_1\Delta Q_i + G_1$

$A_2\Delta y_{i+1} + B_2\Delta Q_{i+1} = C_2\Delta y_i + D_2\Delta Q_i + G_2$

For the eternal conditions

Need a linear relation

Imposed water surface elevation at point 15 be function of time f(t)

$$^{m+1}y_{15}^{n+1} = y(t_{n+1})$$

$$^{m}y_{15}^{n+1} + \Delta y_{15} = y(t_{n+1})$$

A1=0, B1=0, C1=1, D1=0, G1= $y(t_{n+1}) - y_{15} \approx 0 but \neq 0$

If no storage in this device

$$^{m+1}Q_{in}^{n+1} = {}^{m+1}Q_{out}^{n+1}$$

$$^{m+1}Q_{15}^{n+1} = {}^{m+1}Q_{16}^{n+1} \quad \text{or} \quad {}^{m}Q_{15}^{n+1} + \Delta Q_{15} = {}^{m}Q_{16}^{n+1} + \Delta Q_{16}$$

A2=0, B2=1, C2=0, D2=1, G2= $-{}^{m}Q_{16}^{n+1} + {}^{m}Q_{15}^{n+1} \approx 0 but \neq 0$

For any internal de st. venant computational reach, we need 2 linealized equations among $\Delta y_i \Delta y_{i+1} \Delta Q_i \Delta Q_{i+!}$

1 will express mass conservation in some form

1 will express some dynamic condition for y,Q or a relation between them

Consider a rectangular weir

Basic equations:

- Free flowing weir, no downstream influence

$$Q = C_d \, {}^2\!/_3 \sqrt{{}^1\!/_3 \sqrt{2g}} \, (y_{us} - y_{dw})^{3/2}$$

- Flooded Weir

$$Q = C_d \sqrt{2g} \, (y_{us} - y_{ds})^{1/2} (y_{us} - y_{dw})$$

Flooded if $y_{ds} - y_{dw} > \frac{2}{3}(y_{us} - y_{dw})$ [?]

We need to linearlized the weir equations to get A1,B1,C1…

For continuity: $^{m+1}Q_{i}^{n+1} = {}^{m+1}Q_{i+1}^{n+1}$

=>for A2=C2=0, B2=D2=1 G2= $^{m}Q_{i}^{n+1} - {}^{m}Q_{i+!}^{n+1} \approx 0 but \neq 0$

Dynamic equations $^{m}Q_{i+1}^{n+1} = f({}^{m}y_{us}^{n+1}, {}^{m}y_{dw}^{n+1})$

$$^{m}Q_{i+1}^{n+1} + \Delta Q = f({}^{m}y_{us}^{n+1}, {}^{m}y_{dw}^{n+1}) + \frac{\partial f}{\partial y_{us}}|_m \, \Delta y_{us} + \frac{\partial f}{\partial y_{dw}}|_m \, \Delta y_{dw} \ldots\ldots$$

If free flow $\frac{\partial f}{\partial y_{us}} = C_d \frac{2}{3}\sqrt{\frac{1}{3}}\sqrt{2g}\frac{3}{2}(y_{us} - y_{dw})^{1/2}$

$\frac{\partial f}{\partial y_{ds}} = 0$ no influence from ds

If flooded $\frac{\partial f}{\partial y_{us}} = C_d \sqrt{2g}(y_{ds} - y_w)\frac{1}{2}(y_{us} - y_w)^{-1/2}$

$\frac{\partial f}{\partial y_{ds}} \neq 0$

Now final step associate f, $\frac{\partial f}{\partial y_{us}}, \frac{\partial f}{\partial y_{dw}}$ with I, i+1 to get A1,B1,….G1 for double sweep

Case I: flow from i->i+1:

i <-> y_{us} i+1<->y_{ds}

$A_1 = \frac{\partial f}{\partial y_{ds}}|_{^m y_{i+1}^{m+1}} \quad C_1 = -\frac{\partial f}{\partial y_{us}}|_{^m y_i^{m+1}}$

B1=0 D1=1

G1=$^m Q_i^{m+1} - f\left(^m y_i^{n+1}, ^m y_{i+1}^{n+1}\right)$

CaseII: flow from i+1->i:

i <-> y_{down} i+1<->y_{up}

$A_1 = \frac{\partial f}{\partial y_{us}}|_{^m y_{i+1}^{m+1}} \quad C_1 = -\frac{\partial f}{\partial y_{down}}|_{^m y_i^{m+1}}$

B1=0 D1=1

G1=$^m Q_i^{n+1} + f\left(^m y_i^{n+1}, ^m y_{i+1}^{n+1}\right)$

Remarks

As presented here, we are limited to sub critical flow F<1

- Double sweep assume 1 us and 1 ds bc subcritical
  - De st venant equations assume continuous floe ie no
  - Hydrualic jump
    - Put $\frac{\partial}{\partial x}(\alpha \frac{Q^2}{A})$ one can cheat setting afa =0 but you didnt

get exactly answer

- Initial condition must be set $y^0 Q_i^0$ for all i=1,2…

- De st Vanant equation will accept an initial condition which is wrong by computing unsteady adjustment to the consistent situation.

Usually we like to have a steady flow as a starting point for unsteady simulation:

- Give some responsible initial condition (eq depth const)
- Hold b c constant
- Let unsteady equations converge to steady flow (no more changes in y and Q)
- Then stort unsteady simulation

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- Application to pipes
- Branched Networks
- Looped networks → Channels or pipes
- Flood plain storage
- GRDA example

Application to pipes

Flow pipe flow $h_L = f\frac{L}{D}\frac{V^2}{2g}$ => $\frac{h_L}{L} = f\frac{}{D}\frac{V^2}{2g} = S_f$

Mixed full + free surface pipes

(Storm drainnetwork or sewage network during a storm)

Presissman's idea: use free surface tools (de st.Venant equations)

To handle mixed free-pressurised flow

Piezometric levels:

Hydrostatic pressure:

$p = \gamma h$

Elevation of free surface

Specific weight

Presismann slot:

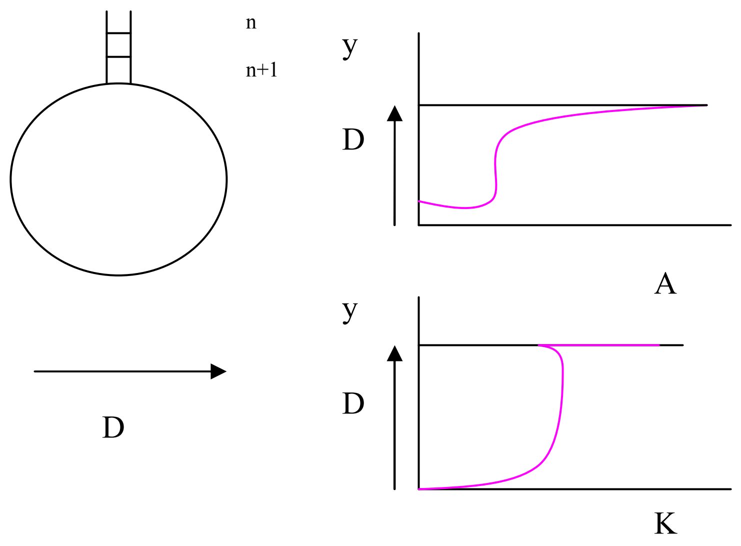
Continuous longitudinal "piezometer"(pressurized surface)

Idea: use a slot that is quite narrow (~a few mm or a few cm)

So that the continuity error it causes is small, but dynamics of full pipe flow are represented

- Area=const for all pipe
- Near-instantaneous propagation of a discharge pertobation

We need to make the slot small enough so that any change in area is small and therefore inflow discharge perturbations must result in ~ same inflow perturbation

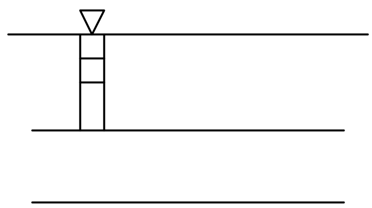


Slot: conveyance don't increase when y increase

Two issues

1) Rapid changes in pressure can cause catastropiz drops in w.s elevation. Iterations are needed in this case
2) Technique can not use for water hammer simulation
   ->Need true full pipe simulation typically method characteristics

manholes



Branched Networks

"Simply connected"
"Tree like"

Basis applying double sweep is:

1) require water continuity at a junction

   $$\sum {}^{m+1}Q_l^{n+1} = 0$$ **no storage in junction**

   K!= of flowpaths links that need at a junction

2) Assume **constant water surface elevation** at all the computational points associate to the junction

   $${}^{m+1}y_l^{n+1} = Y, k = 1, \ldots K$$

At each point on a flow path coming from a boundary condition
We have $\Delta Q = E\Delta y + F$ E F known from the forward sweep
Let i=1be the first computational point on a link coming out of a junction

$${}^{m+1}Q_1^{n+1} = \sum_{k=1}^{K}({}^{m}Q_1^{n+1} + \Delta Q_k) = \sum_{k=1}^{K} {}^{m}Q_k^{n+1} + \sum_{k=1}^{K}(E_k \Delta y_k + F_k)$$

$$\Rightarrow \quad E_1 = \sum_{k=1}^{K} E_k \qquad F_1 = \sum_{k=1}^{K} {}^{m}Q_k^{n+1} + \sum_{k=1}^{K}(E_k({}^{m}y_1^{n+1} - {}^{m}y_k^{n+1}) + F_k) - {}^{m}Q_1^{n+1}$$

- Execute the forward sweep from all bc pts through junctions to a single other bc points
- Use the last boundary condition to apply $\Delta Q, \Delta y$ (as in a single channel)
- Perform return sweep noting eq. $\Delta y_k = \Delta y_1$

Looped similar non-structured coeff matrix

We don't have a simple way to carry E, F sweep through juctions.

Generalized double sweep (Russians): Assume $\Delta Q_i = E_i \Delta y_i + F_i + H_i \Delta y_1$

And $\Delta Q_1 = E_i \Delta y_i + F_i + H_i \Delta y_1$

This leads to any link $\Delta Q_I = E_I \Delta y_I + F_I + H_I \Delta y_1$

$$\Delta Q_1 = E_I^{'} \Delta y_I + F^{'}{}_I + H^{'}{}_I \Delta y_1$$

From algebra manipulation of our F1=0, F2=0 equations

Replate dischanges at each end of a link to W.s elevation changes at each end of a link

Now: Invoke junction continuity sum Q =0

Same water depth at all points

So sum Q=0 and equal y at junction

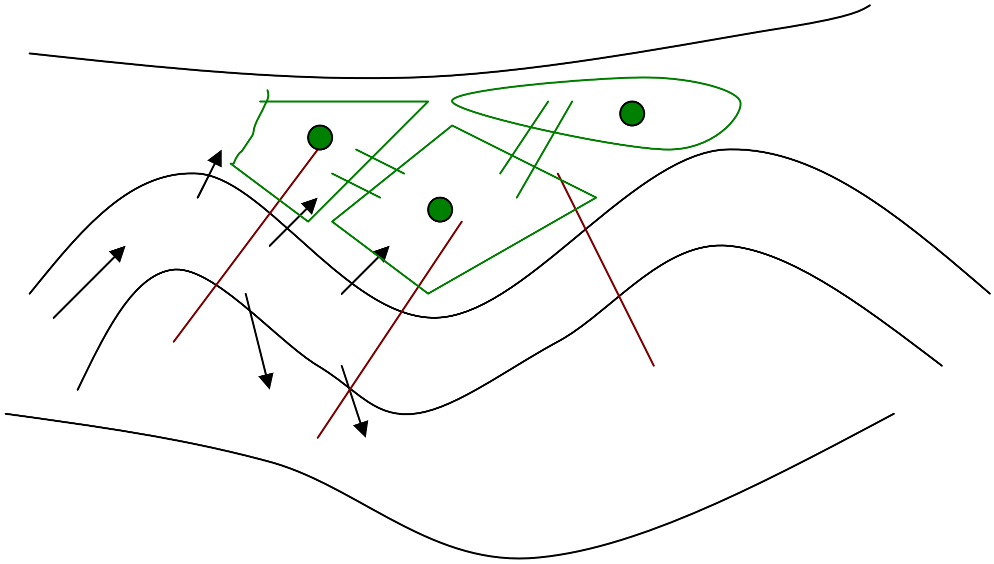
- ⇨ linear equation for each junction involving deta y values at this junction and at junctions to wish it is connected
- ⇨ system of linear equations in deata y at all junctions of model

Computational procedure in one iteration of one time step:

1) perform generalized forward sweep to get E F H and E' F' H' for all comp points
2) Formulate linear system in deta y (nodes) from juctions continuity summation Q=0
3) Solve linear system for deta y nodes (matrix interation)
4) Perform return sweep on all links to get deta Q deta y at computational points.

Flood plain storage

What about networks or channels that are not really one dimetional?



Assume each flood plain cell is a junction

Flood plain cells:

Model of flood plain storage with non inertial equations

A cell has a flat w.s elevation

$$\sum Q = \frac{ds}{dt} = surfaceArea \frac{dy}{dt}$$

Flows from channel to a cell; or between cells

Represented as non lineartial hydraulic Laws (not de st Venant)

Eq Weir equation

Manning – Strickler equation

Culvert equation

Pump str

=> $Q = f(y_{us}, y_{ds})$